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Amplitudes Upon Reflection in Acoustical Experiments

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Abstract

This short note describes some simple acoustical wave experiments that explain the physics of reflection and transmission coefficient formulae for normal incidence P-waves. The experiments help us to understand the mathematical expressions for the reflection and transmission coefficients for displacement and pressure that are frequently used in seismology.

Introduction

Reflection coefficients denote the ratio of the amplitude of a reflected wave to that of an incident wave. These coefficients are derived by solving the boundary conditions for continuity of displacement and stress at an interface. The reflection coefficients for a normally incident P-waves are derived in detail by Robinson and Treitel (1980, 2008) for both displacement and pressure. These equations basically involve the solution of two equations and two unknowns in computing solutions to the boundary conditions.

Reflection coefficient strength is governed by contrast in acoustical impedance, ρv (product of rock density and P-wave velocity). Consider the reflection at an interface between two layers where the wave is passing from layer 1 to layer 2 as shown in Figure 1.

As derived by Robinson and Treitel (1980, 296-329). The reflection coefficient for displacement, R, is given by:

$$R = \frac{\rho_1 v_1 - \rho_2 v_2}{\rho_1 v_1 + \rho_2 v_2} \quad . (1)$$

The reflection coefficient for pressure, r, is given by:

$$r = \frac{\rho_2 v_2 - \rho_1 v_1}{\rho_2 v_2 + \rho_1 v_1} \quad . (2)$$

The transmission coefficients are given by the ratio of the transmitted wave amplitude to the incident wave. Due to the boundary conditions imposing continuity of displacement and pressure, the transmission coefficients for displacement, T, and for the transmission coefficient for pressure, t, are given by:

$$1 + R = T \quad . (3)$$

$$1 + r = t \quad . (4)$$

Therefore, in terms of density and velocity, the transmission coefficient expressions for displacement and pressure become:

$$T = \frac{2\rho_1 v_1}{\rho_1 v_1 + \rho_2 v_2} \quad . (5)$$

$$t = \frac{2\rho_2 v_2}{\rho_2 v_2 + \rho_1 v_1} \quad . (6)$$

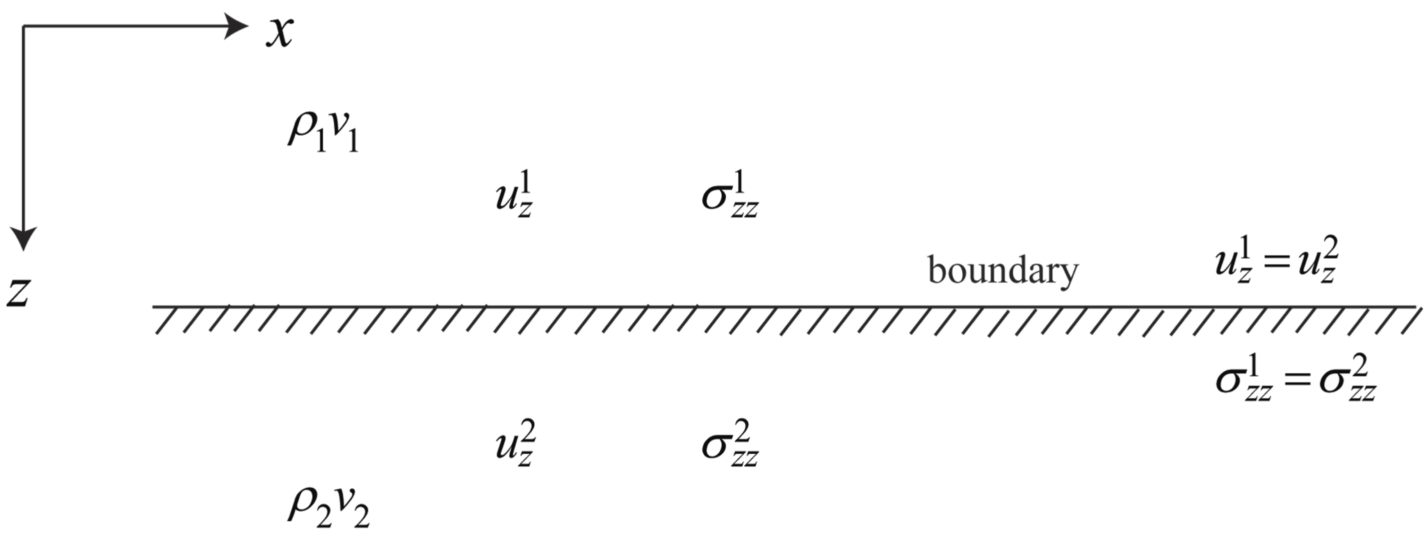

Figure 1. The figure shows a boundary between two layers of contrasting acoustical impedance. The boundary conditions require continuity of the normal displacement and the normal component of stress.

At first glance, the reader may be confused by the difference in the sign of the reflection coefficients for displacement and pressure in equations (1) and (2). Also the numerators of the expressions for the transmission coefficients for displacement and pressure differ. We now explain the difference between these expressions by considering some simple acoustical experiments and show that the above expressions predict the correct physical behaviour of reflected and transmitted waves.

Insight can be gained by realizing that the time derivative of displacement (particle velocity of a wave's amplitude) is a vector quantity measured by a geophone. For a compressional wave (acoustic signal) the sign of a vector is determined by the direction of wave travel. On the other hand, pressure is a scalar quantity whose magnitude is measured by a hydrophone and its sign is independent of direction of wave travel. The different responses of a hydrophone and a geophone provided the central idea for the design of the dual sensor used in multiple attenuation (Robinson and Treitel, 2008).

We shall consider a couple of acoustical experiments which explains why equations (1)-(6) are correct and predict the same physical phenomena.

Acoustical Experiments

Some acoustical experiments which explain the above expressions are the reflections and transmissions of sound waves from closed tubes and open tubes. These experiments, which also can describe the acoustical behaviour of musical instruments, represent the simplest types of experiments where sound waves pass from low impedance to high impedance and vice-versa. This explanation is taken from the classical Berkeley physics book entitled *Waves* by Crawford (1968).

Closed Tube (Low Impedance – High Impedance) Reflections

The reflection of a sound wave from a closed end tube represents the reflection situation of a wave passing from very low to very high acoustical impedance. Consider the compressional part of a sound wave passing through a tube with a closed end. At the end of the tube, the boundary condition requires that the average displacement of air molecules to be permanently zero. Figure 1 describes this situation and the boundary conditions that are described by Crawford's book. We describe the physics of reflected sound and show that the measurements are consistent with the reflection coefficient and transmission formulae shown in equations (1), (2), (5) and (6).

At the closed end of the tube, the average velocity of air molecules is zero. Since the tube is rigid and closed, there is no displacement. That is, the displacement of wave particles is zero. We can envision every air molecule travelling toward the boundary as being hit by an "image molecule" traveling in the opposite direction. (The same "image displacement" argument holds for transverse waves as shown in Crawford (1968), p238-239). Therefore, an incoming compressional wave hitting a fixed rigid boundary gives rise to a compressional wave travelling in the opposite direction. Let us now see how this phenomenon is predicted by the reflection and transmission coefficient formula given previously.

For all practical purposes, we can consider the impedance of air to be nearly zero and the impedance of the closed tube to be very large. In other words, let us consider the case of $\rho_2 v_2 >> \rho_1 v_1$. In such a situation, we see that $R \approx -1$ and $r \approx 1$.

For measuring displacement (or more correctly, its time derivative), we use a geophone. The geophone will measure a compressional arrival travelling in the opposite direction to the incident wave arrival with equal amplitude. Since the geophone is measuring a displacement, a vector quantity, it will measure a reflected arrival with equal amplitude and opposite sign giving a reflection coefficient of -1.

For measuring the pressure of the reflected wave, we use a hydrophone, which is the seismic equivalent of a pressure gauge. The hydrophone measures pressure, a scalar quantity which is insensitive to the direction of wave travel. The reflected compression has equal magnitude to the incident wave travelling in the opposite direction. The pressure reflection coefficient, r, has a value of +1. Both reflection coefficient formulae predict the same physical phenomenon of an incident compression being reflected and giving rise to a compression travelling in the opposite direction.

Let us examine the transmission coefficients for this situation. According to (5), the displacement will have a transmission coefficient of T=0. This value of T agrees with the fact that the boundary of the closed tube is rigid and there is no displacement as a result of the reflection. However, there is certainly a pressure on the end of the closed

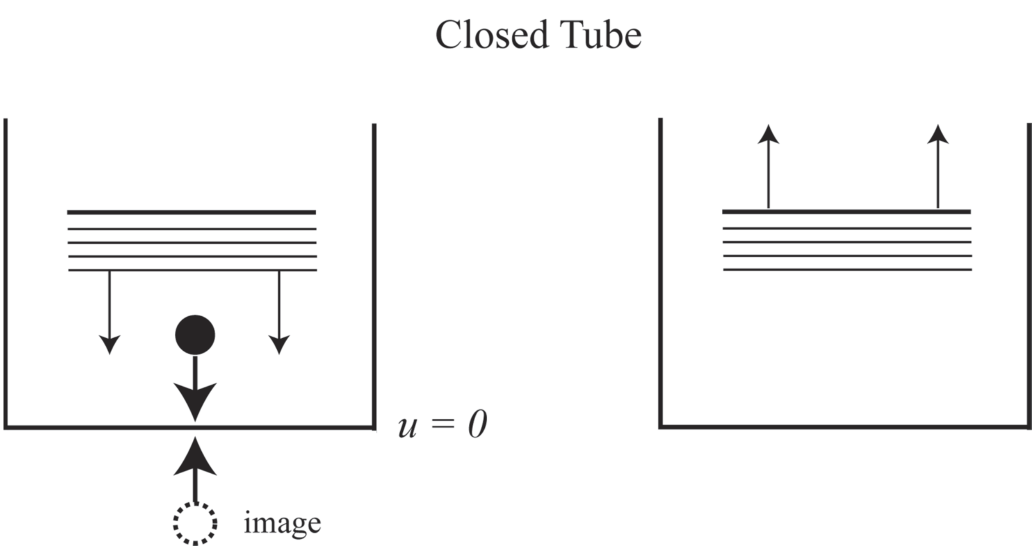

Figure 2. Acoustical experiment for a compressional wave incident on a closed tube. An incident compression (left) gives rise to a reflected compression (right).

tube. The transmission coefficient predicts a value of t=2. To understand this we recall that pressure is force per unit area. Also, we recall that force is equal to momentum change per unit time. If a molecule has mass m and is travelling with velocity v, the momentum before reflection is mv and after reflection is –mv; hence the change of velocity is 2mv. The molecule transfers twice the momentum it would deliver if the incident wave continued down the tube. The transmission coefficient for the pressure change is 2.0.

For the closed tube experiment (low impedance to high impedance transition) we note that the reflection and transmission coefficients for displacement and pressure predict the correct physical behaviour.

Open Tube (High Impedance – Low Impedance) Reflections

We now consider reflections and transmission of acoustic waves for waves travelling from high impedance to low impedance. This is achieved by considering the flow of compressed air out the end of an open tube. This experiment, as shown in Figure 3, is explained lucidly in more detail by Crawford (1968).

At the end of an open tube, the displacement of air molecules (or their velocity) is not constrained to be zero, as in the case of the closed tube. As a compressional wave leaves the entrance to the open tube, the air molecules are not confined to the tube and spread out sideways, and at some distance from the tube (about the distance of the tube's radius), the pressure associated with the compressional wave reaches the equilibrium pressure of the surrounding space. The excessive outflow of air causes a pressure deficit at the open end of the tube, creating a rarefaction. This phenomenon is essentially the Bernoulli effect which relates the flow speed of a fluid to the pressure in the system. As air molecules of a compressional wave speed up at the open end, the pressure decreases. This is the same principle used in design of an airplane wing, where flow over the top of the wing causes reduced pressure and a resulting lift to the wing.

Therefore, in going from a high impedance medium to a low impedance medium as in the case of the open tube, a compressional wave will give rise to a rarefaction travelling in the opposite direction. Lets verify that our reflection and transmission coefficients predict this result. This is the case in (1) and (2), where $\rho_2 v_2 << \rho_1 v_1$, and for the open tube case, we obtain and $R \approx 1$ and $r \approx -1$. This is the correct prediction for our experiment. A rarefaction travelling upward with about the same amplitude as a compression travelling downward and have a reflection coefficient of nearly 1.0. A pressure measuring device such a pressure gauge which is insensitive to direction of wave travel, will produce a reflection coefficient of about -1. These values are predicted by our reflection coefficient formula.

As for the transmission coefficients for open tubes, it may be easiest to recognize that equation (6) gives the correct result. It predicts a value of $t \approx 0$ for the pressure of the transmitted wave. This makes sense since as the wave leaves the tube to an open space, its pressure is greatly reduced. It may be a little more difficult to understand that the displacement's transmission coefficient will be $T \approx 2$ for the case where $\rho_2 v_2 << \rho_1 v_1$. At first glance, this appear to violate conservation of energy, since the amplitude of the transmitted wave increases as the wave travels from high to low impedance. However, the wave's energy is not determined solely by wave amplitude. The energy is given by the square of the wave's amplitude multiplied by a factor that is proportional to the acoustical impedance of the medium (Robinson and Treitel, 1980, p. 298-300). Although the amplitude of the transmitted wave may twice as large as the incident wave, the energy transmitted may be quite small. This phenomenon of large amplitude transmitted waves can be seen in ocean waves as they approach the shoreline (Claerbout, 1976, p.145).

Reflected Amplitudes in the Case of Zero Acoustical Impedance Contrast

Having contemplated the case of reflections from boundaries of low/high and high/low impedance contrast, we consider the case of zero impedance contrast. For normally incident waves in elastic media, our reflection and transmission coefficients in the previous equations predict that there is no reflection (R=r=0) and compete transmission (T=t=1).

If the wave travels at non-normal incidence to the boundary, the effective acoustical impedance is

$$\frac{\rho v}{\cos\theta}$$

(Claerbout, 1976, p. 173). In such cases, we could envision possible reflections for non-normal incidence waves, even if the impedance were constant. If there is a velocity contrast at the boundary, then there will be a refracted wave and $\cos\theta$ will be different on each side of the boundary producing a non-zero reflection coefficient.

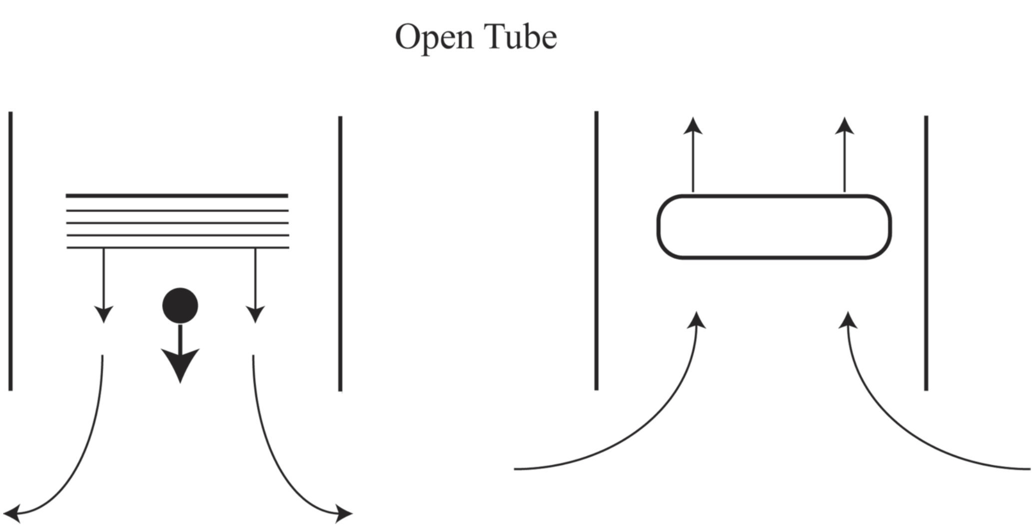

Figure 3. Acoustical experiment for a compressional wave incident on an open tube. An incident compression (left) gives rise to a reflected rarefaction (right).

If we consider anelastic media in which there is differing absorption of seismic energy on each side of a boundary, reflections of non-normally incident waves can occur. This can be seen by examining the reflection coefficients derived by Lines et al. (2008). For such a case, reflections can arise due to changes in acoustical impedance or seismic absorption.

For the anelastic case, the displacement reflection coefficient of equation (1), ρv becomes replaced by $\rho v(1+\frac{i}{2Q})$, where Q is the seismic quality factor (inverse attenuation).

$$R = \frac{\rho_1 v_1\left[1+\frac{i}{2Q_1}\right] - \rho_2 v_2\left[1+\frac{i}{2Q_2}\right]}{\rho_1 v_1\left[1+\frac{i}{2Q_1}\right] + \rho_2 v_2\left[1+\frac{i}{2Q_2}\right]} . \quad (7)$$

In other words, we can obtain seismic reflections from contrasts in the seismic quality factor, Q, even if the impedance, ρv, is constant. Such reflections have been predicted by theory and synthetic seismograms (Lines et al., 2008) and have been measured in lab measurements (Sondergeld, personal communication). Some of the first evidence to demonstrate Q-reflections was given by Bourbie and Gonzalez-Serrano (1983) and by Bourbie and Nur (1984).

Conclusions

There is sometimes confusion in the sign conventions for reflection coefficients, and the sign depends on whether we are measuring displacement (or particle velocity, its time derivative) or pressure. Perhaps the easiest way to understand the reflection coefficients is to consider acoustical experiments for reflections in the cases of increasing impedance or decreasing impedance. This can be understood by referring to the discussions of the physics of waves for closed tubes and open tubes. This short note relates the mathematical equations for reflection and transmission coefficients to the physics of reflected acoustic waves.

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