APPLICATION OF THE FOURIER TRANSFORM TO THE SOLUTION OF THE REVERSE PROBLEM OF GRAVITY AND MAGNETIC SURVEYS

By

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(Translation — N. Rothenburg)

In the quantitative geological interpretation of gravity and magnetic depth various methods of preliminary transformation of the original curves are becoming increasingly more popular.

E. Kogbetliantz [12] applied integral transforms for the depth determination of a bounded prism.

A. A. Popov found the depth to the upper edge of a thin (sheet) bed utilizing the following equation:

$$J(\omega) \approx 2\pi\sigma e^{-\omega h}, \quad (1)$$

where h denotes depth to the upper edge of the bed, σ is the density of the magnetic mass; J (ω) is the Fourier cosine transform of the magnetic field's vertical component of intensity Z calculated according to the formula:

$$J(\omega) = \int_{-\infty}^{\infty} Z(x) \cos \omega x dx.$$

The logarithm of the Fourier cosine transform J (ω) according to (1) is expressed graphically as a straight line the tangent of whose slope is numerically equal to the depth of the bed's upper boundary.

I. G. Klushin determined depth to the upper edge of a fault from the gravity anomaly $\triangle$q curve with the aid of integral transforms [5].

In the present work the Fourier transform pair is considered [8].

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(\omega) e^{-i\omega x} d\omega; \quad (2)$$

$$S(\omega) = \int_{-\infty}^{+\infty} f(x) e^{i\omega x} dx, \quad (3)$$

where f (x) denotes real or complex functions satisfying certain conditions, guaranteeing convergence of the above noted integrals [8]; S (ω) is the Fourier transform of the function f (x); ω is some real variable. In the present article expressions of the Fourier transforms S (ω) are

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obtained for two dimensional bodies whose vertical cross section is limited to a finite number of beds.

Sheets, faults and other two dimensional bodies of angular form are referred to this group. Special templates are presented to facilitate the Fourier transform S (ω) calculation and methods of parameter determination by Fourier transform are worked for geological objectives.

THEORETICAL BASIS OF METHOD

Gravity Surveying

Complex expressions are employed from the third derivatives of the gravitational potential which hold true for certain two dimensional bodies of constant density, with cross section restricted to a finite number of beds [11].

$$U_{zzx} - iU_{\Delta x} = \sum_{k=1}^{n} \frac{A_k}{x - \tau_k}, \tag{4}$$

where A_k is some complex constant depending on the magnitude of the excess density and geometrical dimensions of the body, $\tau_k = x_k + ih_k$; x_k, h_k are the co-ordinates of the apices of the prism's cross section.

On the basis of formulas (3) and (4), we obtain

$$S_2(\omega) = \int_{-\infty}^{\infty} (U_{xzx} - iU_{\Delta x}) e^{i\omega x} dx = \sum_{k=1}^{n} A_k \int_{-\infty}^{\infty} \frac{1}{x - \tau_k} e^{i\omega x} dx. \tag{5}$$

According to theory [5], the integral appearing in formula (5) equals

$$\int_{-\infty}^{\infty} \frac{e^{i\omega x}}{x - \tau_k} dx = 2\pi i e^{i\omega\tau_k}. \tag{6}$$

Using (5) and (6) we find the expression of the Fourier transform for the third derivative of the gravitational potential.

$$S_2(\omega) = \int_{-\infty}^{\infty} (U_{xzx} - iU_{\Delta x}) e^{i\omega x} dx = 2\pi i \sum_{k=1}^{n} A_k e^{i\omega\tau_k}. \tag{7}$$

Integrating (7) once by parts and placing the exterior term of the integral equal to zero we obtain the expression of the Fourier transform for the second derivative of the gravitational potential.

$$S_1(\omega) = \int_{-\infty}^{\infty} (U_{xz} - iU_{\Delta}) e^{i\omega x} dx = -\frac{2\pi}{\omega} \sum_{k=1}^{n} A_k e^{i\omega\tau_k}. \tag{8}$$

For calculation of the Fourier transform from the first derivative of the gravitational potential it is necessary to utilize the following relation.

$$(U_{xz} - iU_{\Delta}) = \frac{\partial}{\partial x}(U_z + iU_x). \tag{9}$$

Substituting in (9) and (8) and integrating once by parts we obtain, assuming the exterior term of the integral equals zero.

$$S_0(\omega) = \int_{-\infty}^{\infty} (U_z + iU_x) e^{i\omega x} dx = -\frac{2\pi i}{\omega^2} \sum_{k=1}^{n} A_k e^{i\omega\tau_k} . \tag{10}$$

Relations (7) (8) and (10) may be simplified using the following equation [2].

$$\begin{aligned} \int_{-\infty}^{\infty} U(x, 0) \cos \omega x dx &= -\int_{-\infty}^{\infty} V(x, 0) \sin \omega x dx, \\ \int_{-\infty}^{\infty} U(x, 0) \sin \omega x dx &= \int_{-\infty}^{\infty} V(x, 0) \cos \omega x dx, \end{aligned} \tag{11}$$

where U and V are the real and imaginary parts of the complex expression. On the basis of (11) the relations (7) (8) and (10) may be reduced to the following form:

$$\begin{aligned} S_2(\omega) &= \int_{-\infty}^{\infty} (U_{xzx} - iU_{\Delta x}) e^{i\omega x} dx = 2 \int_{-\infty}^{\infty} U_{xzx} e^{i\omega x} dx = \\ &= 2\pi i \sum_{k=1}^{n} A_k e^{i\omega\tau_k} , \end{aligned} \tag{12}$$

$$\begin{aligned} S_1(\omega) &= \int_{-\infty}^{\infty} (U_{xz} - iU_{\Delta}) e^{i\omega x} dx = 2 \int_{-\infty}^{\infty} U_{xz} e^{i\omega x} dx = \\ &= -\frac{2\pi}{\omega} \sum_{k=1}^{n} A_k e^{i\omega\tau_k} , \end{aligned} \tag{13}$$

$$\begin{aligned} S_0(\omega) &= \int_{-\infty}^{\infty} (U_z + iU_x) e^{i\omega x} dx = 2 \int_{-\infty}^{\infty} U_z e^{i\omega x} dx = \\ &= -\frac{2\pi i}{\omega^2} \sum_{k=1}^{n} A_k e^{i\omega\tau_k} , \end{aligned} \tag{14}$$

MAGNETIC SURVEYING

For the case of the derivatives of the magnetic potential it is necessary to make use of the relations established by Poisson [11].

$$H - iZ = -\frac{Ji}{k\sigma} (U_{xz} - iU_{\Delta}) e^{i\gamma} \tag{15}$$

and likewise

$$\frac{\partial}{\partial x}(H - iZ) = -\frac{Ji}{k\sigma}\frac{\partial}{\partial x}(U_{xz} - iU_{\Delta})e^{i\gamma}, \tag{16}$$

where γ is the inclination angle of the magnetisation intensity vector. Employing (12) and (13) we obtain

$$S_2(\omega) = \int_{-\infty}^{\infty}\frac{\partial}{\partial x}(H - iZ)\, e^{i\omega x}\, dx = 2\pi i\sum_{k=1}^{n} B_k e^{i\omega\tau_k} \tag{17}$$

$$S_1(\omega) = \int_{-\infty}^{\infty}[(H - iZ)\, e^{i\omega x}\, dx = -\frac{2\pi}{\omega}\sum_{k=1}^{n} B_k e^{i\omega\tau_k}, \tag{18}$$

where
$$B_k = A_k\left(-\frac{J}{k\sigma}e^{i\gamma}\right).$$

As a result of recent extensive development in aeromagnetic methods, we calculate the Fourier transform from the $\triangle T$ curve. For this we make use of the relation which holds true for anomalies of small intensity (in the order of the first 1000γ) [7, 9]

$$\frac{\partial H_1}{\partial x} - i\frac{\partial Z_1}{\partial x} = \left(\frac{\partial H}{\partial x} - i\frac{\partial Z}{\partial x}\right)(\sin J_0 - i\cos\delta\cos J_0), \tag{19}$$

where J_0 denotes the angle of normal magnetic inclination; δ is the angle between the profile and the magnetic meridian; $Z_1 = \triangle T$; H_1, is a function combining with $Z_1 = \triangle T$

$$\frac{\partial H_1}{\partial x} = -\frac{\partial Z_1}{\partial z}; \quad \frac{\partial Z_1}{\partial x} = \frac{\partial H_1}{\partial z}.$$

We denote * $\sin J_0 - i\cos\delta\cos J_0 = A_0$, (20)

Then equation (19) takes the form

$$\frac{\partial H_1}{\partial x} - i\frac{\partial Z_1}{\partial x} = A_0\left(\frac{\partial H}{\partial x} - i\frac{\partial Z}{\partial x}\right). \tag{21}$$

On the basis of (17), (18) and (21) we have

$$S_2(\omega) = \int_{-\infty}^{\infty}\frac{\partial}{\partial x}(H_1 - iZ_1)e^{i\omega x}\, dx = 2\pi i\sum_{k=1}^{n} C_k e^{i\omega\tau_k}, \tag{22}$$

$$S_1(\omega) = \int_{-\infty}^{\infty}(H_1 - iZ_1)e^{i\omega x}\, dx = -\frac{2\pi}{\omega}\sum_{k=1}^{n} C_k e^{i\omega\tau_k}, \tag{23}$$

where
$$C_k = A_0 B_k.$$

Using equations (11) we bring the expressions (17), (18), (22) and (23) to the following form:

$$S_2(\omega) = \int_{-\infty}^{\infty} \frac{\partial}{\partial x}(H - iZ)\, e^{i\omega x}\, dx = -2i \int_{-\infty}^{\infty} \frac{\partial Z}{\partial x} e^{i\omega x}\, dx =$$

$$= 2\pi i \sum_{k=1}^{n} B_k e^{i\omega\tau_k}, \tag{24}$$

$$S_1(\omega) = \int_{-\infty}^{\infty} (H - iZ)\, e^{i\omega x}\, dx = -2i \int_{-\infty}^{\infty} Z e^{i\omega x}\, dx =$$

$$= -\frac{2\pi}{\omega} \sum_{k=1}^{n} B_k e^{i\omega\tau_k}, \tag{25}$$

$$S_2(\omega) = \int_{-\infty}^{\infty} \frac{\partial}{\partial x}(H_1 - iZ_1)\, e^{i\omega x}\, dx = -2i \int_{-\infty}^{\infty} \frac{\partial Z_1}{\partial x} e^{i\omega x}\, dx =$$

$$= 2\pi i \sum_{k=1}^{n} C_k e^{i\omega\tau_k}, \tag{26}$$

$$S_1(\omega) = \int_{-\infty}^{\infty} (H_1 - iZ_1)\, e^{i\omega x}\, dx = -2i \int_{-\infty}^{\infty} Z_1 e^{i\omega x}\, dx =$$

$$= -\frac{2\pi}{\omega} \sum_{k=1}^{n} C_k e^{i\omega\tau_k}. \tag{27}$$

DETERMINATION OF BODY PARAMETERS FROM FOURIER TRANSFORMS

Let us turn to the depth determination of the disturbing object in the case of a complicated geological section. As an example formula (24) is utilized and we assume that the point, $\tau_1 = x_1 + ih_1$, is always located closer to the earth's surface in comparison with other points τ_k i.e. we assume $h_1 < h_2$; $h_1 < h_3 \ldots$; $h_1 < h_n$. Then noting that $\tau_k = x_k + i\,h_k$ the expression (24) may be written down in the following form:

$$S_2(\omega) = 2\pi i \sum_{k=1}^{n} B_k e^{i\omega x_k}\, e^{-\omega h_k}.$$

From here it is not difficult to satisfy the condition that for large ω there is the approximation:

$$S_2(\omega) \approx 2\pi i B_1 e^{i\omega x_1} e^{-\omega h_1}.$$

We take the modulus of both sides of the equation presented above, after which we find

$$|S_2(\omega)| \approx 2\pi |B_1| e^{-\omega h_1}, \tag{28}$$

where

$$|S_2(\omega)| = \sqrt{[S_2'(\omega)]^2 + [S_2''(\omega)]^2},$$

$$S_2'(\omega) = \int_{-\infty}^{\infty} \frac{\partial Z}{\partial x} \cos \omega x dx,$$

$$S_2''(\omega) = \int_{-\infty}^{\infty} \frac{\partial Z}{\partial x} \sin \omega x dx.$$

Taking the logarithm of relation (28) we obtain the formula which is shown to be suitable for large values of ω

$$h_1 \approx -\frac{\ln |S_2(\omega_2)| - \ln |S_2(\omega_1)|}{\omega_2 - \omega_1}. \tag{29}$$

Transforming equation (25) and (10) also, we find

$$h_1 \approx -\frac{\ln [\omega_2 |S_1(\omega_2)|] - \ln [\omega_1 |S_1(\omega_1)|]}{\omega_2 - \omega_1}, \tag{30}$$

$$h_1 \approx -\frac{\ln [\omega_2^2 |S_0(\omega_2)|] - \ln [\omega_1^2 |S_0(\omega_1)|]}{\omega_2 - \omega_1}. \tag{31}$$

Analogous expressions were obtained by V. K. Ivanov who demonstrated that they remain valid for the arbitrary relationship of varying body thickness [3]. Equation (31) was employed by I. G. Klushin [5] for determining depth to the upper edge of a fault from the Δq curve. The basic inadequacy of formulas (29) to (31) stems from the fact that for their application it is necessary to calculate the Fourier transform for large values of ω. At the same time it is known that for large ω values the accuracy of Fourier transform calculation sharply decreases for an observed curve complicated by errors. In this connection some special procedures for determination of depth h_1 are presented below free of the indicated shortcoming.

Thick Bed

Let us examine a thick dipping bed with a horizontally arranged upper edge. We further assume that within the bed the magnetic intensity vector is arbitrarily positioned relative to the dip of the bed. In this case according to (25) we find:

$$S_1(\omega) = -2i \int_{-\infty}^{\infty} Z e^{i\omega x} dx = -\frac{2\pi}{\omega} (B_1 e^{i\omega\tau_1} + B_2 e^{i\omega\tau_2}). \tag{32}$$

Here $\tau_1 = x_1 + ih$ (where x_1, h are the co-ordinates of the left angular point of the bed); and $\tau_2 = x_2 + ih$ (where x_2, h are the co-ordinates of the right angular point of the bed).

Taking into account that $B_1 = -B_2 = B$ [11] we obtain

$$S_1(\omega) = -\frac{2\pi}{\omega} Be^{-\omega h} \left(e^{i\omega x_1} - e^{i\omega x_2}\right). \tag{33}$$

We take the modulus of both sides of equation (33)

$$|S_1(\omega)| = 2\sqrt{\left(\int_{-\infty}^{\infty} Z\cos\omega x dx\right)^2 + \left(\int_{-\infty}^{\infty} Z\sin\omega x dx\right)^2} =$$

$$= \frac{2\pi}{\omega}|B|e^{-\omega h}\sqrt{(\cos\omega x_1 - \cos\omega x_2)^2 + (\sin\omega x_1 - \sin\omega x_2)^2} =$$

$$= \frac{4\pi}{\omega}|B|e^{-\omega h}\sin\omega b, \tag{34}$$

where $2b = x_2 - x_1$, the horizontal thickness of the bed.

Further, we multiply (34) again by the variable transform ω

$$|S_1(\omega)|\omega = 4\pi|B|e^{-\omega h}\sin\omega b. \tag{35}$$

With the aid of the substitution [1]

$$Y = \frac{|S_1(\omega_{k+2})|\omega_{k+2}}{|S_1(\omega_k)|\omega_k}, \tag{36}$$

$$X = \frac{|S_1(\omega_{k+1})|\omega_{k+1}}{|S_1(\omega_k)|\omega_k}$$

the expression (35) is reduced to the equation of a straight line

$$Y = B_0X - A_0, \tag{37}$$

where

$$B_0 = 2e^{-\omega_0 h}\cos\omega_0 b, \tag{38}$$

$$A_0 = e^{-2\omega_0 h}.$$

Here $\omega_k = k\omega_0$, the value of the variable Fourier transform taken through equal intervals ω_0; $|S_1(\omega_k)|$ is the modulus value of the Fourier transform, taken at point ω_k.

Formulas for depth determination of the upper edge of the bed and its horizontal thickness can be derived from (38)

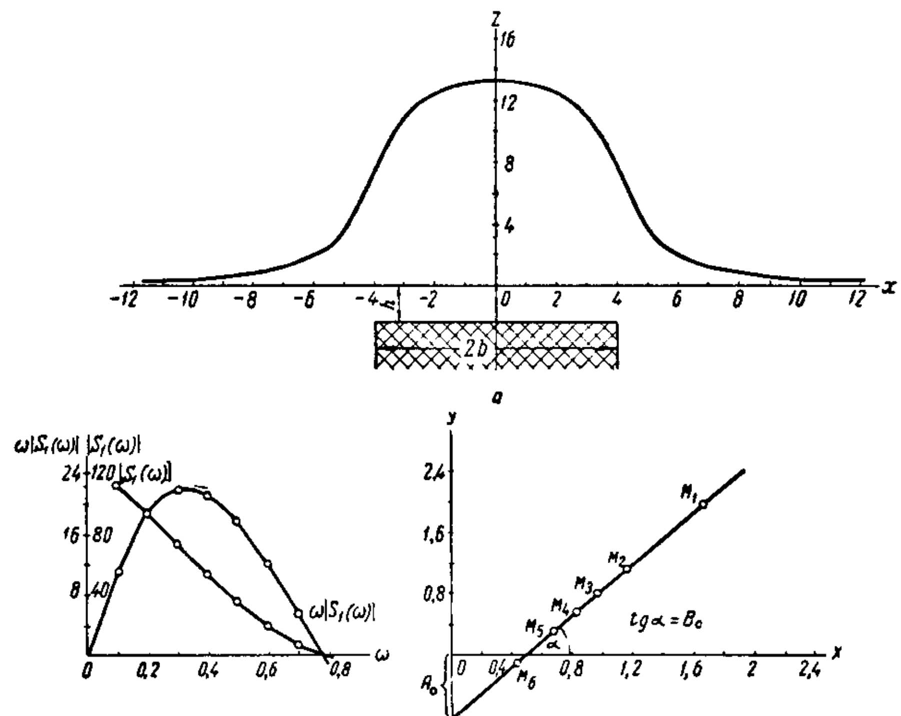


FIG. 1. Thick Bed.

a—curve of the vertical component of the anomalous magnetic field Z; b—curve of the Fourier transform modulus $|S_1(\omega)|$ and curve of the product $\omega|S_1(\omega)|$; c—straight line graph.

$$h = \frac{1}{\omega_0} \ln \frac{1}{\sqrt{A_0}}, \tag{39}$$

$$b = \frac{1}{\omega_0} \arccos \frac{B_0}{2\sqrt{A_0}}. \tag{40}$$

It should be noted that for a bed whose horizontal thickness 2b is less than the depth to its upper boundary "h," the points M_k (Fig. 1c) of the straight line found in the method stated above are located very close to each other that it is difficult to draw a straight line (37).

In this case we should turn to the thin bed formula.

Thin Beds

For thin beds the value b is small. Therefore, the approximation

$$\sin \omega b \approx \omega b, \tag{41}$$

can be used, which together with (34) gives

$$|S_1(\omega)| = 4\pi b |B| e^{-\omega h} \tag{42}$$

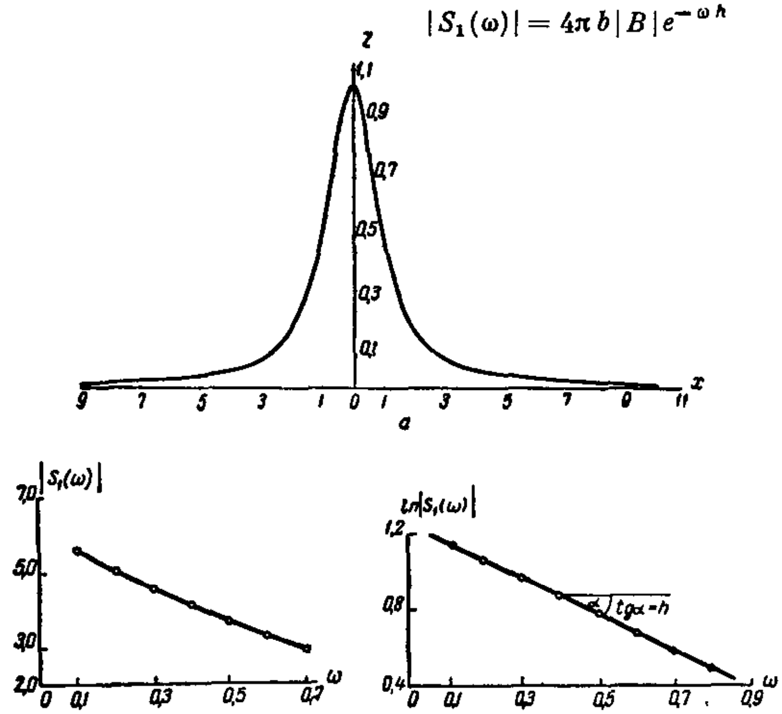


FIG. 2. Thin Bed.

a—Curve of the vertical component for anomalous field of intensity Z, b—Curve of modulus $|S_1(\omega)|$, c—Logarithm of Fourier transform modulus $\ln|S_1(\omega)|$.

The logarithm of the Fourier transform modulus $|S_1(\omega)|$ is expressed graphically as a straight line (Fig. 2c) with tangent of the slope angle numerically equal to the depth "h."

Vertical Scarp (Fault)

Let us consider a vertical scarp and assume that the direction of the magnetization intensity vector is arbitrary. Formula (32) is also applicable in this case in which the substitution $\tau_1 = x_1 + ih_1$, and $\tau_2 = x_1 + ih_2$ should be made where x_1 and h_1 are the co-ordinates of the upper apex (angular point) of the scarp, and x_1, h_2 are the co-ordinates of the lower angular point of the vertical scarp (fault).

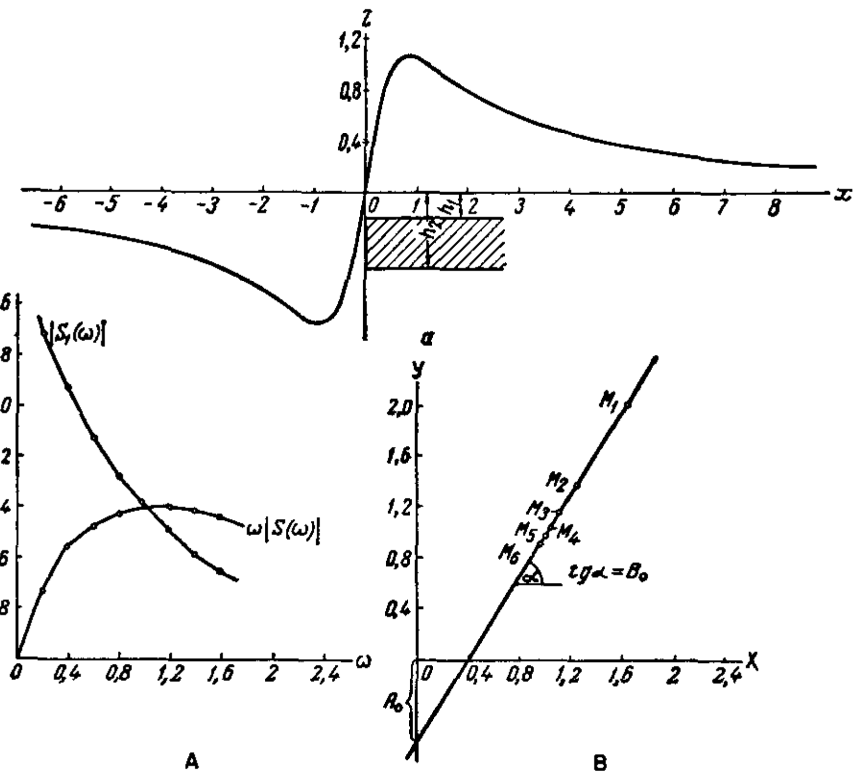


FIG 3. Vertical Scarp (fault)

a—Curve of vertical intensity component of anomalous magnetic field Z; b—Curve of the Fourier transform modulus $|S_1(\omega)|$ and product curve $\omega|S_1(\omega)|$; c—Graph of straight line (equation (45)).

The modulus of the expression $|S_1(\omega)|$ will equal

$$|S_1(\omega)| = \frac{-2\pi B}{\omega}\left(e^{-\omega h_1} - e^{-\omega h_2}\right). \tag{43}$$

We multiply (43) by the variable ω

$$|S_1(\omega)|\,\omega = -2\pi B(e^{-\omega h_1} - e^{-\omega h_2}). \tag{44}$$

The expression (44) is transformed by substitution of (36) into the equation for a straight line.

$$Y = B_0 X - A_0, \tag{45}$$

where

$$B_0 = e^{-\omega_0 h_1} + e^{-\omega_0 h_2}; \tag{46}$$

$$A_0 = e^{-\omega_0 (h_1 + h_2)}.$$

From (46) formulas can be readily obtained for depth determination to the median line of the scarp and to its upper edge.

$$\frac{h_1 + h_2}{2} = \frac{1}{\omega_0} \ln \frac{1}{\sqrt{A_0}},$$

$$h_1 = \frac{h_1 + h_2}{2} - \frac{1}{\omega_0} \operatorname{arc\,ch} \frac{B_0}{2\sqrt{A_0}}.$$

Attention is drawn to the fact that in all formulas presented above the natural logarithm is used. Determination of the constants A_0 and B_0 is explained in Fig. 3c.

CALCULATION OF FOURIER TRANSFORMS

The Fourier transform (3) is broken down into the cosine transform $S'(\omega)$ and the sine transform $S''(\omega)$

$$S'(\omega) = \int_{-\infty}^{+\infty} f(x) \cos \omega x dx,$$

$$S''(\omega) = \int_{-\infty}^{+\infty} f(x) \sin \omega x dx. \tag{47}$$

Let us examine the influence of the normal field selection on the magnitude of these intervals. We assume that the normal field is incorrectly chosen and is $f'(x) = a$. Then (47) is written in the form

$$\int_{-\infty}^{+\infty} a \sin \omega x dx = \lim_{L \to \infty} \int_{-L}^{+L} a \sin \omega x dx = -a \lim_{L \to \infty} \left. \frac{\cos \omega x}{\omega} \right|_{-L}^{+L},$$

$$\int_{-\infty}^{+\infty} a \cos \omega x dx = \lim_{L \to \infty} \int_{-L}^{+L} a \cos \omega x dx = a \lim_{L \to \infty} \left. \frac{\sin \omega x}{\omega} \right|_{-L}^{+L}.$$

It is questioned whether these integrals revert to zero. This is possible only in the case when

$$L = \pm \frac{k\pi}{\omega}. \tag{48}$$

For such a selection the limit of the normal field integration does not show up in the value of the Fourier transform. Considerable difficulty occurs in finding a numerical value of the integral (47) for large values of ω. We therefore use Filon's method [10], specially worked for these cases. Filon proposed the following formula for calculation of the sine transform.

$$S''(\omega) = \Delta' [\alpha f(0) + \beta R_{2k} + \gamma R_{2k-1}], \tag{49}$$

where Δ' is the distance between points for which the values f (x) are known; $\omega = \frac{\pi}{4n\Delta'}$ is some real variable sometimes called the frequency; 2n is the number of intervals included in the range $|o, \frac{\pi}{2\omega}|$;

$$R_{2k} = \sum_{k=0}^{n} \left[f(2k\Delta') \sin \frac{k\pi}{2n} - \frac{1}{2} f(2\pi\Delta') \right]; \tag{50}$$

$$R_{2k-1} = \sum_{k=0}^{n} \left[f(2k\Delta' - \Delta') \sin \frac{2k-1}{2n} \pi \right]; \tag{51}$$

α, β, γ are Filon's coefficients (see Table 1).

n	$\theta = \frac{\pi}{4n}$, radians	$\theta = \frac{\pi}{4n}$, degrees	α	β	γ
5	$\pi/20$	9	0,000171700	0,6693340	1,3300464
4	$\pi/16$	11°15′	0,000396306	0,6721400	1,3278890
3	$\pi/12$	15	0,000789700	0.6756270	1,3242170
2	$\pi/8$	22°30′	0,002642050	0,6863380	1,3128910
1	$\pi/4$	45	0,019710800	0,7352204	1,2528780

TABLE 1.

Substituting (50) and (51) in (49) we obtain

$$S''(\omega) = \int_0^{\pi/2\omega} f(x) \sin \omega x dx = \Delta' \sum_{k=0}^{2n} d_k f_k, \tag{52}$$

where d_k are the coefficients whose values are presented in Table 2; f_k — values of the function at points separated by the distance Δ'.

Knowing the coefficients d_k for the range $(O, \frac{\pi}{2\omega})$, it is possible to find the values of the d_k coefficients for other intervals, taking account of the periodic character of the changing function $\sin \omega x$.

The most difficult operation in the calculation by formula (52) is the multiplication. One may be relieved of this if the multiscaled template is used [4]. The template is superimposed on the $f(x)$ curve whose Fourier transform is required. Readings of C_k'' are taken at intersections of the template scale with the $f(x)$ curve, taking account of the sign shown on the template. The value of the Fourier transform $S''(\omega)$ is calculated according to the formula:

$$S''(\omega) = \frac{\Delta MN}{10} \sum_k C_k'', \tag{53}$$

where M is the horizontal scale coefficient, i.e. the number of units measured along 1*cm.* of the curve's abscissa axis.

N is the vertical scale coefficient, i.e. the number of units measured along 1*cm.* of the curve's ordinate axis. Δ is the distance along the x axis between the template scales measured in centimeters (the previous value Δ' is related by $\Delta' = \Delta M$). The number "10" in the indicated formula (53) is introduced because of the fact that the numerical scale of the template is magnified tenfold. The C''_k coefficients corresponding to the boundary of the interval

$$\left[(K-1)\frac{\pi}{2\omega}, K\frac{\pi}{2\omega}\right] \text{ и } \left[K\frac{\pi}{2\omega}, (K+1)\frac{\pi}{2\omega}\right],$$

are exaggerated twofold, since they appear twice in the integration through two adjacent intervals. The template is constructed for ω values equal to $1\omega_0$, $2\omega_0$, . . .

$$\omega_0 = \frac{\pi}{4n\Delta'} = \frac{\pi}{4n\Delta M} = \frac{\pi}{4 \cdot 5 \cdot 1{,}57M} = \frac{0{,}1}{M} (n = 5). \tag{54}$$

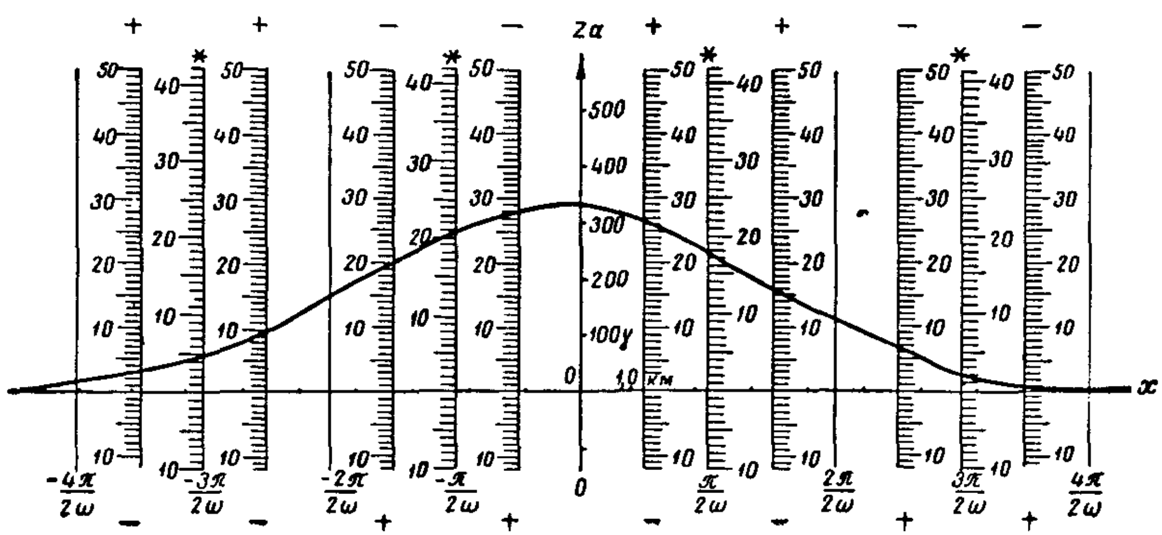
It is necessary to trace these templates onto a transparency, completing the scale for the $\left[\frac{\pi}{2\omega}, \frac{2\pi}{2\omega}\right]$, $\left[\frac{2\pi}{2\omega}, \frac{3\pi}{2\omega}\right]$ range, etc. in order to obtain a template whose length extends to about 20 cm. on one side.

Fig. 4 shows the layout for the template $S''(7\omega_0)$.

As an example we show the calculation of the Fourier sine - transform $S''(7\omega_0)$ of the Z curve presented in Fig. 4. The sum of the readings of the corresponding template scale equals $\Sigma C''_k = 0 + 3.5 + 5.0 + 9.5 - 20.0 - 20.5 - 28.0 + 26.5 + 18.0 + 16.5 - 6.8 - 2.3 - 1.0 + 0 = 0.4$. In the sum referred to above, the last figure zero is written down in

n	Δ	d_0	d_1	d_2	d_3	d_4	d_5	d_6	d_7	d_8	d_9	d_{10}
5	1,5708	0,00017	0,20802	0,20701	0,60384	0,39379	0,94047	0,54198	1,18507	0,63717	1,31369	0,33497
4	0,9817	0,000396	0,25906	0,25722	0,73528	0,47528	1,10411	0,62098	1,30240	0,33610	—	—
3	0,8726	0,000789	0,34273	0,33781	0,93637	0,58512	1,27910	0,33781	—	—	—	—
2	0,9817	0,00264	0,50242	0,48531	1,21295	0,34316	—	—	—	—	—	—
2	0,7854	0,00264	0,50242	0,48531	1,21295	0,34316	—	—	—	—	—	—
1	1,3089	0,019711	0,88592	0,36761	—	—	—	—	—	—	—	—
1	1,1219	0,019711	0,88592	0,36761	—	—	—	—	—	—	—	—
1	0,9817	0,019711	0,88592	0,36761	—	—	—	—	—	—	—	—

TABLE 2.



$\omega = 7\omega_0$; $\Delta = 1,1219$; $S^*(\omega) = \frac{\Delta NM}{10}\sum c_k$.

FIG. 4. Example of sine Transform Calculation by Template.

order to emphasize the fact that the readings should be completed on the zero scale. This is necessary for fulfillment of condition (48).

The Fourier sine-transform is found by formula (53).

$$S''(7\omega_0) = \frac{\Delta M \cdot N}{10} \sum C_k'' = \frac{1{,}1219 \cdot 1{,}0 \cdot 100}{10} \times 0{,}4 = 4{,}49.$$

In accordance with formula (54) the value of the variable transform equals

$$\omega = 7\omega_0 = 7\frac{0{,}1}{M} = 7 \times \frac{0{,}1}{1\ \text{км}} = 0{,}7\ \text{км}^{-1}.$$

For calculation of the Fourier cosine-transform for the same Z curve the co-ordinate origin of the template is shifted to the right by an interval of $\frac{\pi}{2\omega}$.

An example of the Fourier cosine-transform calculation is shown in Fig. 5.

The sum of the readings from the template scale equals
$\Sigma c_k = \frac{1}{2}\, 0.5 + 2.5 - 9.3 - 12.0 - 19.8 + 27.0 + 24.0 + 26.0 - 15.7 - 9.4 - 7.0 + 0.9 + \frac{1}{2} 0 = 7.45$.
In the calculation of the Fourier cosine-transform the template readings should be completed on the scale marked by asterisk to fulfill condition (48). The reading on the last scale is multiplied by 1/2. The numerical value of the cosine-transform is found from the formula

$$S'(7\omega_0) = \frac{\Delta MN}{10} \sum C_k' = \frac{1{,}1219 \cdot 1 \cdot 100}{10} \times 7{,}45 = 83{,}58.$$

The value of the variable transform $\omega = 7\omega_0$ as in the previous case equals 0.7 km^{-1}.

PRACTICAL EXAMPLE

The curve of the magnetic field's vertical component of magnetisation Z is shown in profile in Fig. 6a. Values of the Fourier transform moduli were calculated for this curve with the aid of the template for intervals along the ω axis equal to $\omega_0 = \frac{0{,}1}{M} = \frac{0{,}1}{0{,}25\ \text{км}} = 0.4\ \text{Km}^{-1}$ (Fig. 6b). A graph of the natural logarithm of the transform moduli was plotted against the variable transform which is represented as a straight line in agreement with the case discussed previously (see section on "Thin Beds").

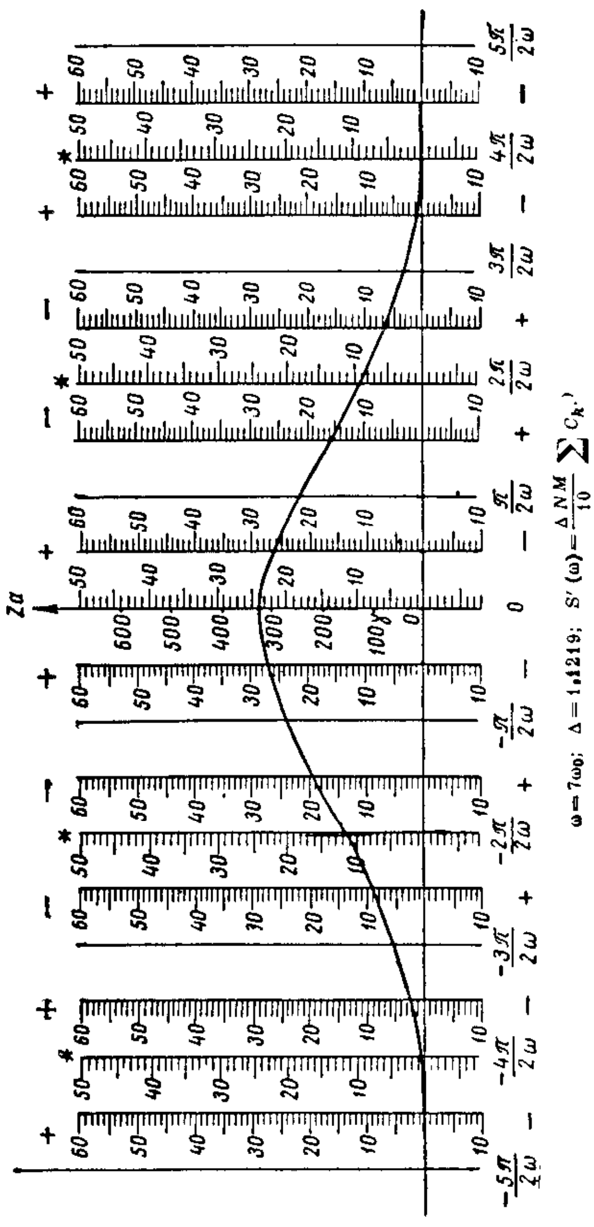


FIG. 5. Example of Fourier cosine - transform calculation by template.

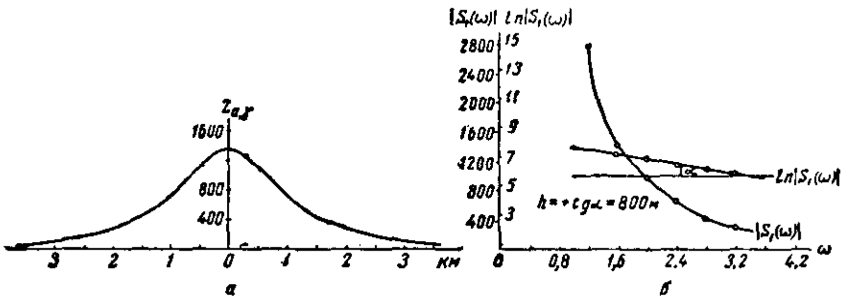


FIG. 6. Practical Example

a—Curve of vertical intensity component of anomalous magnetic field Z. b—Curve of modulus $|S_1(\omega)|$; and logarithm of Fourier transform modulus $\ln|S_1(\omega)|$.

A depth to the upper edge h = 800 m was found from the tangent of the slope of the line $\ln|S_1(\omega)|$ which agrees within 6.6% of the depth of the causative object obtained from borehole data (h = 750 m).

In conclusion we extend our appreciation to Engineer A. A. Groznov who checked the method on fifty magnetic and gravity anomalies.

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