

A NOTE ON SYNTHETIC VIBROSEIS SWEEP GENERATION

by

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In reading through the article "The Vibroseis System of Seismic Mapping" by Mr. R. L. Geyer (J.C.S.E.G. 6, 1, 1970) I came across some equations (see Fig. 9 in Geyer's article) which did not follow algebraically one from the other, namely:

$$S = \text{Re}(\text{Rect}(t/T) \exp(2\pi i(f_0 t + kt))) \quad (1)$$

$$S = t/T \cos(2\pi(f_0 t + kt^2/2)) \quad (2)$$

$$= \cos(2\pi f_0 t) \cos(\pi kt^2) - \sin(2\pi f_0 t) \sin(\pi kt^2) \quad (3)$$

These equations do not follow logically one from the other and in looking at them I was led to reconstruct that section of his article.

I would like then to start with the expression given by Klauder et al⁽¹⁾ for a general linearly-swept-frequency waveform

$$y(t) = \text{Rect}(t/T) \exp(2\pi i(f_0 t + kt^2/2)) \quad (4)$$

$$\text{where } \text{Rect}(t/T) = \begin{cases} 1 & \text{if } T/2 \leq t \leq T/2 \\ 0 & \text{if } T/2 > t > T/2 \end{cases} \quad (5)$$

with T = the duration of the pulse

and where f_0 and k are still to be defined but f_0 is a reference frequency and k is a rate of increase of frequency with time. Now the real part of (4) is clearly

$$\text{Rect}(t/T) \cos 2\pi(f_0 t + kt^2/2) \quad (6)$$

and not as given in (2). We may replace the modulating function $\text{Rect}(t/T)$ by any suitable function, let us call it $A(t)$ and leave it undefined for the moment except to say that it exists on the interval $-T/2 < t < T/2$. We may write, for the general linearly-swept-frequency signal

$$y(t) = A(t) \cos 2\pi(f_0 t + kt^2) = A(t) \cos 2\pi f(t) \quad (7)$$

with $f(t) = f_0 t + kt^2/2$ and then define the instantaneous frequency

$$f_i(t) = \partial f / \partial t = f_0 + kt. \quad (8)$$

We note that in defining $\text{Rect}(t/T)$ in the original equation (6) we assumed that the sweep was of length T in time, extending over the

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interval $-T/2 < t < T/2$ and we shall use this interval to sweep the signal in frequency from a low frequency f_1 at $t = -T/2$ to f_2 at $t = T/2$, $f_1 < f_2$.

Thus equn. (8) at $t = -T/2$ reads

$$f_1 = f_0 - kT/2 \quad (9)$$

while at $t = T/2$ it reads

$$f_2 = f_0 + kT/2 \quad (10)$$

so that $k = (f_2 - f_1)/T$ — and $f_0 = (f_2 + f_1)/2$

Then equation (7) becomes

$$y(t) = A(t) \cos(\pi t/T (f_2 (t+T) - f_1 (t-T))) \quad (11)$$

It should be noted that at $t = T/2$ the function f has the magnitude

$$f = t/2T(f_2 (t+T) - f_1 (t-T)) \rightarrow T/8 (3f_2 + f_1) \quad (12)$$

and does not equal f_2 ; it is only the instantaneous frequency which is equal to f_2 at $t = T/2$.

One may now compute the expression $y(t)$ by letting $\Delta t =$ the sample period of the data and let $t = n\Delta t$, $T = m\Delta t$ so that

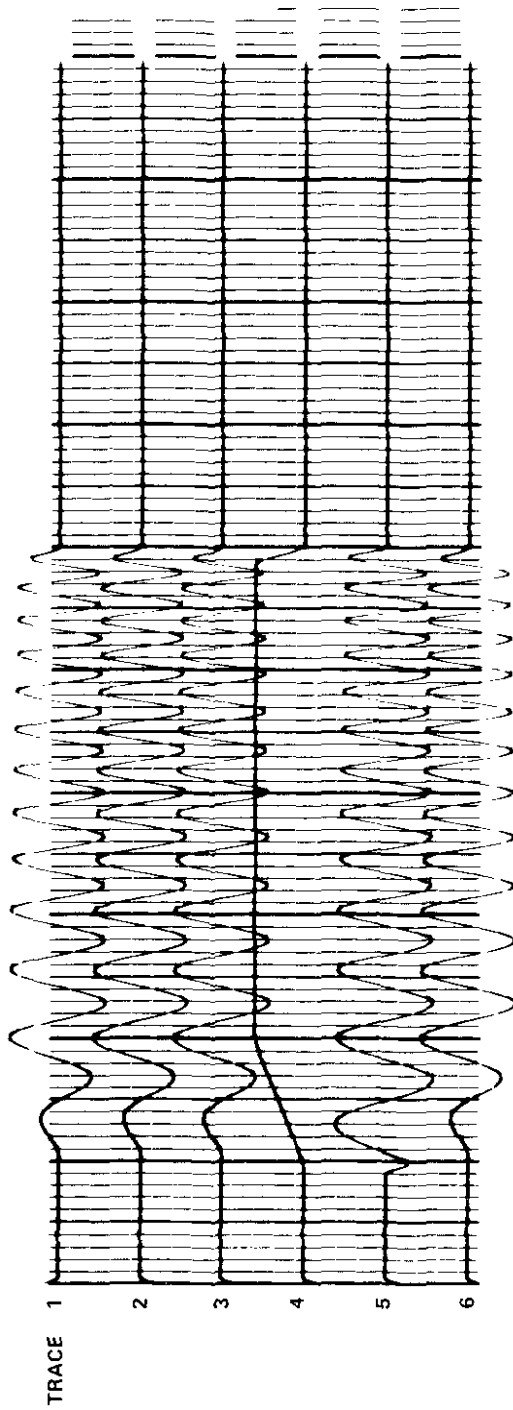
$$y(n\Delta t) = A(n\Delta t) \cos (\pi n/m (f_2 (n+m) - f_1 (n-m))\Delta t) \quad (13)$$

which of course is to be computed for $-m/2 \leq n \leq m/2$.

Some typical sweeps generated by use of (13) are shown below, and the modulating function $A(t)$ is also shown, it has been chosen to simulate the rise and fall time of a Vibroseis sweep.

References:— (1) "The theory and Design of Chirp Radars"

Bell System Tech. Journ. 39, 4, 1960



Trace 5 unmodulated L.S.F. pulse $f_1 = 10$, $f_2 = 40$ Hz.

Trace 4 modulating envelope.

Traces 1, 2, 3, 6 final modulated L.S.F. pulse.

(Note. Decrease in overall amplitude of the sweep at higher frequencies is due to plotter attenuation)

$\Delta t = 4$ m.s. Sweep duration 500 m.s.

Ramp on 100 m.s. Ramp off 10 m.s. Delay 100 m.s.