A NOTE ON THE APPLICATION OF WIENER MULTICHANNEL DECONVOLUTION†

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ABSTRACT

The concept of multichannel Wiener deconvolution is investigated. The algorithm for this multichannel filter is outlined, and the problem associated with the use of identical input channel data is discussed. Since source wavelet estimates determine the multichannel filter input, a brief comparison of techniques used in wavelet estimation is presented. Methods for determining the optimum length of a multichannel filter are also mentioned.

The following short note outlines an application of digital multi-channel Wiener filtering to the problem of seismic deconvolution. Methods and applications of multichannel filtering have been outlined by Treitel (1970). Several Fortran programs for multichannel data analysis have been given in a book by Robinson (1967b). Also, a comparison between multichannel and single channel seismic deconvolution has been given by Davies and Mercado (1968).

Treitel (1970) gives a derivation of the equations which define the multichannel filter coefficients. These coefficients, $\underline{\underline{f}}(t)$, satisfy the following system of normal equations for a filter of length m.

$$(\underline{\underline{f}}(0)\ \underline{\underline{f}}(1)\ \dots\ \underline{\underline{f}}(m)) \begin{bmatrix} \underline{\underline{r}}(0) & \underline{\underline{r}}(1) & \dots & \underline{\underline{r}}(m) \\ \underline{\underline{r}}(-1) & \underline{\underline{r}}(0) & \dots & \underline{\underline{r}}(m-1) \\ \vdots & & \ddots & \vdots \\ \underline{\underline{r}}(-m) & \underline{\underline{r}}(-m+1) & \dots & \underline{\underline{r}}(0) \end{bmatrix} = (\underline{\underline{g}}(0)\ \underline{\underline{g}}(1) \dots \underline{\underline{g}}(m))$$

In the above formulation, $\underline{\underline{f}}(t)$, $\underline{\underline{r}}(t)$ and $\underline{\underline{g}}(t)$ all represent matrices. The sizes of these matrices are determined by the number of input channels and output channels. The matrix $\underline{\underline{f}}(t)$ contains elements $f_{ij}(t)$ which operate on the jth input channel to give a contribution to the ith output channel. The matrix $\underline{\underline{r}}(t)$ represents the autocorrelation matrix for the input process at lag t, while the matrix $\underline{\underline{g}}(t)$ represents the cross-correlation matrix of the input process with the desired output process at lag t. The complete mathematical definitions of $\underline{\underline{f}}(t)$, $\underline{\underline{r}}(t)$ and $\underline{\underline{g}}(t)$ are given by Treitel (1970). Using the assumption of ergodicity, the values of $\underline{\underline{f}}(t)$ and $\underline{\underline{g}}(t)$ are given by the following relations for the case of k input channels.

$$\underline{\underline{r}}(t) = \begin{bmatrix} \sum_s X_1(t+s)\, X_1(s) & \sum_s X_1(t+s)\, X_2(s) & \dots & \sum_s X_1(t+s)\, X_k(s) \\ \sum_s X_2(t+s)\, X_1(s) & \sum_s X_2(t+s)\, X_2(s) & \dots & \sum_s X_2(t+s)\, X_k(s) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_s X_k(t+s)\, X_1(s) & \sum_s X_k(t+s)\, X_2(s) & \dots & \sum_s X_k(t+s)\, X_k(s) \end{bmatrix}$$

$$\underline{\underline{g}}(t) = \begin{bmatrix} \sum_s X_1(t+s)\, Y_1(s) & \sum_s X_1(t+s)\, Y_2(s) & \dots & \sum_s X_1(t+s)\, Y_\ell(s) \\ \sum_s X_1(t+s)\, Y_1(s) & \sum_s X_2(t+s)\, Y_2(s) & \dots & \sum_s X_2(t+s)\, Y_\ell(s) \\ \vdots & \vdots & \ddots & \vdots \\ \sum_s X_k(t+s)\, Y_1(s) & \sum_s X_k(t+s)\, Y_2(s) & \dots & \sum_s X_k(t+s)\, Y_\ell(s) \end{bmatrix}$$

Where $(X_1(t), X_2(t), \dots, X_k(t))$ represents the inputs on each of the k channels at time t, and $(Y_1(t), Y_2(t), \dots, Y_\ell(t))$ represents the desired output values on each of the ℓ output channels at time t. In the example given here, $k = 2$ and $\ell = 1$.

The fact that $\underline{\underline{r}}(t)$ is in the form of a Toeplitz matrix allowed Wiggins and Robinson (1965) to obtain a recursive solution to the equations defining a multichannel Wiener filter. Their recursive solution is

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THE WIGGINS – ROBINSON RECURSIVE SOLUTION FOR THE MULTICHANNEL WIENER FILTER

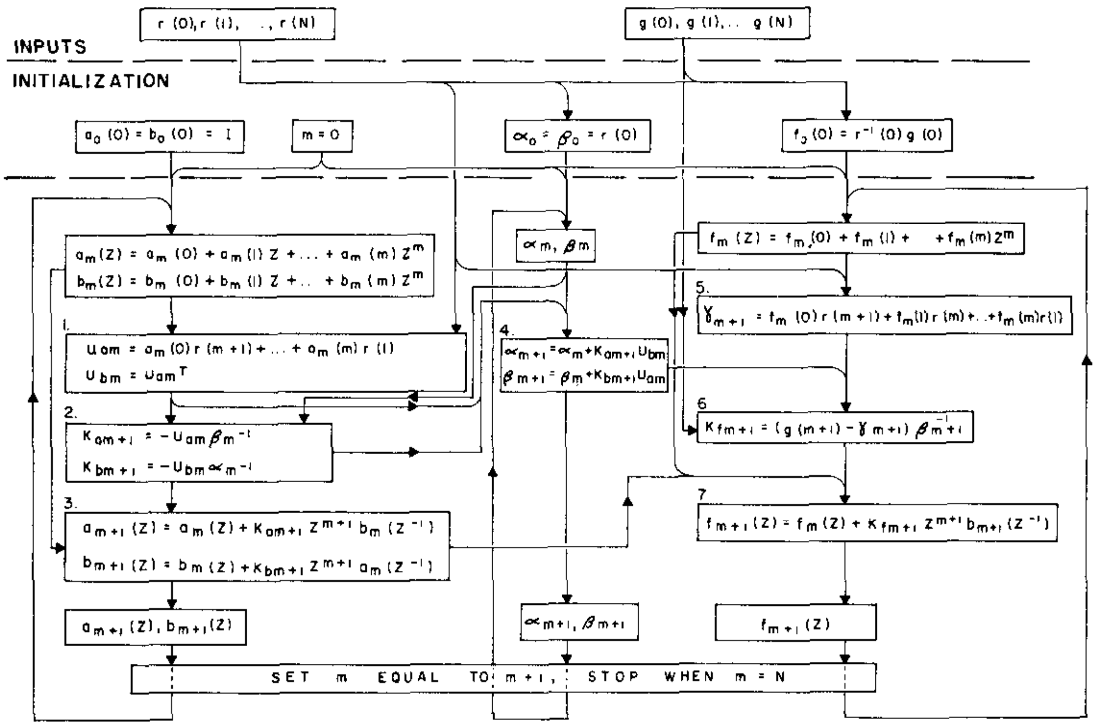

Fig. 1. This flow diagram of the recursive solution to the multichannel Wiener filter follows the diagram given by Wiggins and Robinson (1965) as well as a later description given by Robinson (1967b). The polynomials $a_m(z)$ and $b_m(z)$ represent z transforms of the prediction error and hindsight error sequences. The sizes of the matrix coefficients of these z transforms are determined by the number of input and output channels. In the convention used here for the filter coefficients, $f_m(k)$, k denotes the time index and the subscript m refers to the iteration number in the recursive process.

essentially an extension of the Levinson algorithm from the single channel case. A flow diagram of this solution, which follows the diagram given in Wiggins and Robinson (1965) and the description in Robinson (1967b) is shown in Figure 1. By using this method, an inverse filter for the seismic source may be designed once the source wavelet estimates for each input trace are obtained. Convolution of this multichannel inverse filter with the input seismic traces yields an estimate of the impulse response for a layered earth. However, the source wavelet estimates which are the multichannel input to this program must not be identical. As pointed out by Robinson (1967b), initialization of the recursive process for the filter design requires that $\underline{\underline{f}}(0) = \underline{\underline{g}}(0)\,\underline{\underline{r}}^{-1}(0)$, where $\underline{\underline{f}}(0)$ is the filter matrix at $t=0$, $\underline{\underline{g}}(0)$ is the crosscorrelation matrix of the input with the desired output at zero lag, and $\underline{\underline{r}}^{-1}(0)$ is the inverse of the autocorrelation matrix at zero lag. If all inputs to the filter are identical then $\underline{\underline{r}}(0)$ is a matrix with equal elements and its inverse does not exist. This problem of the autocorrelation matrix being singular has been discussed by Galbraith and Wiggins (1968).

In the synthetic example shown in Figure 2, the use of the Wiener multichannel filter for deconvolution was studied under ideal conditions. A seismic source wavelet was synthesized by using a z transform of the form $(-1.10+z)^{2}(1.75+z)^{12}$ and removing the average from the resulting time series. In order to make the autocorrelation matrix nonsingular, a small amount of white noise was added to the wavelet

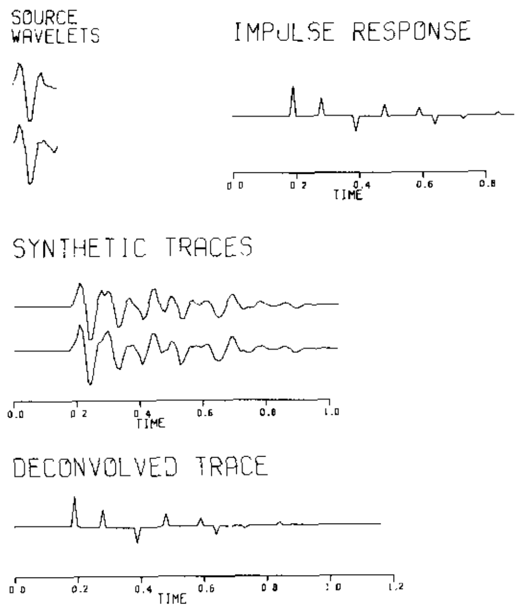

Fig. 2. This synthetic example exhibits the performance of the multichannel Wiener deconvolution filter under ideal conditions. When convolved with the synthetic traces, a multichannel filter of length 14 gives a deconvolved trace which almost duplicates the impulse response.

to give slightly different source wavelets for different input channels. Multiplication of the diagonal elements of $\underline{\underline{r}}(0)$ by a number slightly greater than one is an equivalent method of adding white noise to the source wavelet estimate. The use of slightly different gains on the filter input channels represents a third method of avoiding singularities in the autocorrelation matrix. The source wavelets used in this example were convolved with a given impulse response to yield synthetic seismic traces. When the actual source wavelets are used in the design of an inverse filter, the convolution of this filter with the synthetic traces yields a deconvolved trace which almost duplicates the impulse response. The length of the filter was chosen to be one less than the length of the source wavelet. The multichannel filter gives the correct solution to our problem. This behaviour of the filter agrees with the statement made by Treitel (1975) concerning the performance of the multichannel filter. Treitel points out that the error for the multichannel filter becomes smaller as the number of filter points increases and the error decreases to zero whenever $M = \frac{N-1}{K-1}\Big|_{\text{Mod } 1}$, where M is the filter length, K is the number of input channels and N is the input length. It should be noted that in the single channel case, where K = 1, the Wiener filter length must become infinite in order to obtain ideal performance. Also, it should be pointed out that numerical problems with the multichannel filter may arise if the filter length is taken to be much larger than the length of the input.

Although the results of these idealized tests of the multichannel Wiener filter are encouraging, the test was conducted under the unrealistic assumption that the source wavelets were known. It should be realized that the performance of this deconvolution method is only as good as the source estimation.

There are several methods of estimating the seismic source wavelets in a single channel manner. These wavelet estimates for each trace provide the inputs used in the design of a multichannel deconvolution filter. One of the well known methods of source wavelet estimation is the Wold-Kolmogorov factorization technique. The details of this method are described in an important paper by Robinson (1967a). In using this technique, two assumptions are made.

1. The source wavelet is assumed to be minimum delay.

2. The impulse response for the seismic trace is assumed to be a purely random or white noise series. This condition results in the source autocorrelation being identical to the trace autocorrelation.

Since the impulse response is not a true white noise series, the problem in using Wold-Kolmogorov factorization lies in estimating the source autocorrelation from the trace autocorrelation. Window functions may be used to suppress contributions to the trace autocorrelation at lags much larger than the suspected length of the source wavelet.

A technique which also involves using the trace autocorrelation as an approximation to the source autocorrelation is the Wiener-Levinson double inverse method. In this method, the estimated source autocorrelation values are substituted into the normal equations in order to determine an inverse filter. This inverse filter is itself inverted by again using the normal equations to obtain a source wavelet estimate. In using the Wiener-Levinson double inverse method, the minimum delay assumption was also made since the desired filter outputs were zero delay spikes.

By combining theory and experiment, Ricker (1953) examined the form of seismic wavelets caused by a dynamite explosion. The use of the Ricker wavelet is widespread, and Rice (1962) deconvolved seismic traces by using the Wiener filter and the assumption that the source wavelets were symetric Ricker wavelets. However, Rice admits that problems do arise from lack of information about the source wavelets. In marine seismology, there is additional wavelet information due to the direct observation of the source pulse.

A method which does not require the assumption of a minimum delay wavelet or the assumption that the impulse response be a white noise process is the method of homomorphic deconvolution. This technique has recently been applied by Ulrych (1971) and Ulyrch et al (1972) as a means of estimating seismic wavelets for earthquakes. Homomorphic deconvolution may also hold considerable promise for estimating source wavelets for exploration data.

Figure 3 shows an example of the source estimates for a synthetic seismic trace using Wold-Kolmogorov factorization, the Wiener-Levinson double inverse method, and the method of homomorphic deconvolution.

The trace used in this test was synthesized by convolving the impulse response of Figure 2 with the mixed delay source wavelet shown in Figure 3. (This source wavelet is the wavelet of Figure 2 with no white noise added). The autocorrelation used in the Wold-Kolmogorov and Wiener-Levinson estimates consists of the trace autocorrelation multiplied by a Parzen window of length 20. We see that although the trace autocorrelation is a good estimate of the source wavelet autocorrelation, the minimum delay assumption gives both wavelet estimates a different character from that of the actual source wavelet.

The homomorphic deconvolution estimate was obtained by short pass filtering the trace cepstrum. The details of cepstrum calculations have been outlined by Ulrych (1971). In this case the trace cepstrum was calculated after using a weighting factor of 0.98 on the trace. Since the source cepstrum cannot be completely separated from the trace cepstrum by a short pass filter, the third lobe of the homomorphic deconvolution estimate is not quite the same as that of the actual wavelet.

Using the trace of Figure 3 and the first trace of Figure 2, homomorphic deconvolution estimates were used in applying multichannel deconvolution. A truncated average of the estimates is shown in Figure 4. In this case, using the average of the wavelet estimates as input and multiplying the diagonal of $\underline{\underline{r}}(0)$ by 1.05 provides a better deconvolution than using the individual source wavelet estimates as input. The resulting deconvolution is shown for the cases where the desired output is a spike at delays of 2 units and 5 units respectively. These spiking positions correspond to the first peak and first trough of the estimated wavelets. Optimum spiking positions can be monitored by use of Simpson's sideways recursion which is described by Wiggins and Robinson (1965).

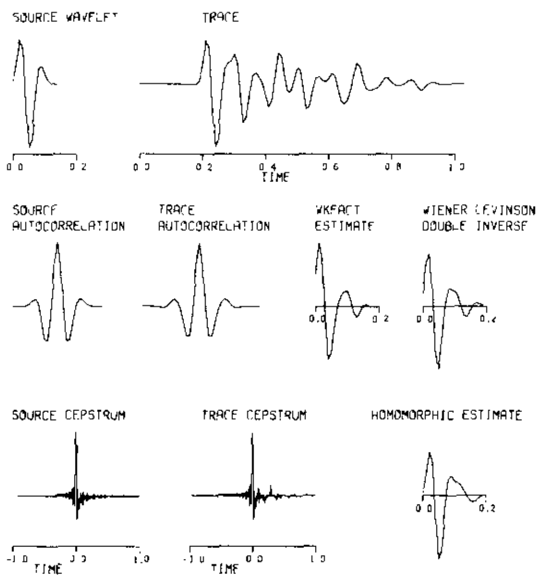

Fig. 3. A comparison of source wavelet estimates found by the Wold-Kolmogorov factorization method (WKFACT), the Wiener-Levinson double inverse method, and the method of homomorphic deconvolution. The zero amplitude positions for the estimated wavelets are given by the axes positions.

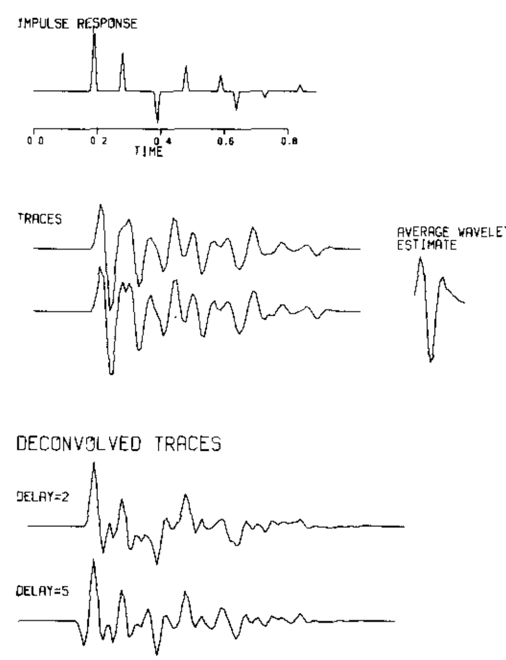

Fig. 4. Multichannel deconvolution resulting from the use of source wavelet estimation is shown for the cases where the desired outputs are spikes at delays of two units and five units.

In conclusion, it would seem that multichannel deconvolution involves two basic problems. The first problem involves the determination of multichannel inputs by source wavelet estimation. As pointed out previously, the filter inputs must be sufficiently different in order to avoid problems in defining the inverse of $\underline{\underline{r}}(0)$. The second problem involves designing the proper multichannel inverse filter. A criterion for determining the length of the multichannel Wiener has been given by Treitel (1975). Numerical difficulties with recursive solutions to these deconvolution problems sometimes arise, and it is advisable to monitor the normalized mean square error as a function of filter length in order to pick a desirable filter length. Fryer et al (1973) determine the optimum inverse filter length for multichannel time series by using the final prediction error statistics of autoregressive models. This may represent an alternative approach to the problem of determining inverse filter lengths.

The purpose of this note has been to outline certain aspects of multichannel deconvolution which may not be immediately apparent. Hopefully the ideas presented in this note will encourage further investigations of multichannel seismic deconvolution and source wavelet estimation.

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