ERROR ANALYSIS IN AUTOMATIC DATA REDUCTION OF POTENTIAL FIELD DATA†

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ABSTRACT

An attempt is made to estimate error bounds in the collection of potential field data using analytical means and to compare these results with observed error distributions. A summary of correction methods is listed along with a critique of each in the light of the error analysis and some possible techniques for utilizing this method are discussed.

INTRODUCTION

Automatic data reduction of potential field data has become the norm in most exploration projects. Offshore seismic programs lend themselves to the inexpensive capture of both gravity and magnetic data in digital format and digital data collection of aeromagnetic data has proven its effectiveness. Since the data are collected in digital format it is common for the user to specify automatic recovery and adjustment of the data with the final product being profiles and maps prepared by digital computers and plotters. Automated analysis routines such as second derivative and downward continuation are inexpensive and useful aids to the interpretation of the data. Automatic location devices appear to have resolved many of the problems encountered in Potential Field surveys and explorationists tend to refer to the maps, profiles and analyses as being accurate representations of the geological phenomena measured.

ERROR ANALYSIS

In most sciences, a single measurement of a physical quantity is treated as a statistical sample of the quantity measured and is usually associated with its measure of dispersion, or probability of being within certain bounds.

This may be stated simply as:

$$V_m = Vo + E$$

where: V_m = measured value
Vo = actual value
E = error in measurement.

If many estimates may be made of Vo as $V_m, V_{m+1}, V_{m+2}, --- V_{m+n}$, then estimates may be made of E and the behavior of E. It is simple to assume that, if E is a randomly distributed value, then as n becomes large, $V_m \rightarrow Vo$ since we would expect the errors to have a mean of zero. It is also possible to arrive at a measure of dispersion (the standard deviation, S), to give us a probability that $Vo - S \leq V_m < Vo + S$.

In field geophysics it is rarely possible to make many estimates of the same physical value. Therefore E remains a statistical mystery and the confidence which may be placed in a single measurement becomes very subjective. Also, to complicate matters, E is not a single value but a combination of values.

At this point, we should review some basic principles of error analysis as they

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refer to the geophysical problem and then examine some examples to gain an appreciation of the effects on the solution of geological problems.

The basic principles of error estimation are simple:

If $Y = x_1 + x_2 + x_3 + \text{---}\ x_i$

and $\hat{x}_i$ = measured or estimated value of x_i

$$\hat{x}_i = x_i + e_i$$

Then $\hat{Y}_i = (x_1 + e_1) + (x_2 + e_2) + \text{---}$

$$(x_i + e_i)$$

and the error in $\hat{Y} = (e_1 + e_2 + \text{---}\ e_i)$

For example, if the corrected value of gravity is given by

$$G_o = G_i + C_E + C_L$$

where: G_i = observed gravity

C_E = Elevation correction

C_L = Latitude correction

and the instrument error is .1% (over 50 mgals) (e_i)

the elevation error is 1% (over 100 meters) (e_E)

the latitude error is 1% (over 100 minutes) (e_L)

then % error in G_o = .1 + 1 + 1 = 2.1%

ERROR ANALYSIS OF MARINE GRAVITY

Now we shall look at an application of these principles in an actual exploration situation. In marine gravity, which we shall use for this example, the function for calculating the corrected gravity G_o is as follows:

$G_o = G_i$ + Eotvos correction + latitude correction

$$= G_i + C_\varepsilon + C_L$$

And $\hat{G}_o = (G_i + e_i) + (C_\varepsilon + e_\varepsilon) + (C_L + e_L)$

Since we cannot take replicate measurements of G_o in an ordinary survey we cannot gain estimates of e_i, e_ε, or e_L. However, we can examine each in the light of differential calculus and gain insight into the range of each e.

1. e_i = the instrument error. This value is usually quantified by the manufacturer and has a well defined dispersion which is implicit in the published "accuracy" of the instrument, e.g. "± .01 mgl".

Instrument Drift error, which is also implicit in e_i is usually compensated to a greater or lesser degree by returning to a previously measured station and distributing the drift over time. However, in Marine Gravity, the location of the previously measured station and its value of G_o is usually in question because of all its measured parameters.

2. e_ε = the error in the EOTVOS correction. This value deserves considerable attention since it is the largest in magnitude and is a combination of several other measurements. The Eotvos correction, C_ε, is given by

$$C_\varepsilon = \frac{(R_\phi + h)}{R_\phi^2} \cdot (2V_\phi V_{east} + V^2)$$

where: R_ϕ = Radius of the earth at latitude ϕ

V_ϕ = Speed of rotation of the surface of the earth at latitude ϕ

V = Ship speed

V_{east} = Eastward component of the ships speed

h = height above sea level.

In practice h = o, and R (mean radius of earth) is substituted for R_ϕ.

Then $\frac{R_\phi + h}{R_\phi^2} = \frac{1}{R}$

Using the appropriate constants

$$C_\varepsilon = 7.503\ V \cos\phi \sin\alpha + 0.00415\ V^2$$

where: C_ε = the Eotvos correction

and α = Ships heading in degrees from N.

Differentiating the above with respect to heading and velocity: $dC_\varepsilon = \frac{\partial C_\varepsilon}{\partial \alpha} d\alpha + \frac{\partial C_\varepsilon}{\partial V} dV$

where: $\frac{\partial C_\varepsilon}{\partial \alpha} = 7.503\ V \cos\phi \cos\alpha$

and: $\frac{\partial C_\varepsilon}{\partial V} = 7.503 \cos\phi \sin\alpha + 0.00830\ V$

This expression allows us to calculate the error in Eotvos Correction given errors in speed and heading.

An absolute example (e.g. disregarding the distribution of heading and speed errors) would be as follows:

The uncertainty in C_L = 1.305 mgs./mile or .082 mgs.

Using the additive properties of errors, the error now is 4.09 + .082 = 4.172. Thus our single point estimate of the Eotvos

Area	— East Coast
Latitude	— 43° 38' (assume no error in Latitude)
Sample Interval	— 1 minute
Location Fix Interval	— 6 minutes

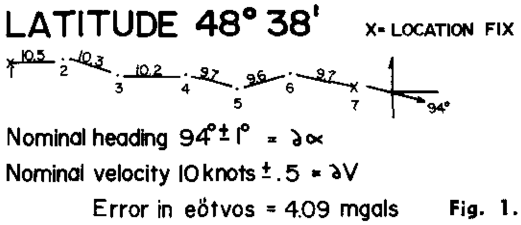

Fig. 1.

x = Fix

• — Reading Location
Nominal heading between fixes = 94°
Heading between station 3-4 = 93°
Velocity between fixes = 10.0 knots
Velocity between stations 3-4 = 9.5 knots

Then

$e_V = 0.5$ knots $= dV$ $\quad e_H = 1° = d\alpha$

$dC_\varepsilon = (7.503\ V \cos\phi \cos\alpha)\ 1.0 + (7.503 \cos\phi \sin\alpha + 0.0083\ V)\ 0.5$

Error in Eotvos = 4.09 mgals

If we reduce the instantaneous heading error to 0.50° then the error in Eotvos correction is still 2.19 mgs. These are very large numbers compared to the magnitude of the anomalies measured.

3. e_L = error in Latitude Correction.

Let us consider that the navigation system gives a location accuracy defined by a circle of 100 meters with a probability of 75%:

error is ± 4.17 mgals for an instantaneous heading error of 1° or 2.27 mgals for an instantaneous heading error of 0.5°.

This assumes also a flat gravitational field. If the field has a strong gradient then the uncertainty in position may also introduce a separate error which is a function of the gradient and direction.

Since we have no estimates of the dispersion and therefore the probability of the heading and velocity errors we must accept this estimate of error as the error bounds for the given heading, speed and location errors. Of course, if we suspect these values are larger we must change limits.

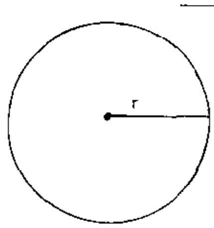

Fig. 2.

• Estimated position.

r = radius of circle within which position may actually be with a probability of 75%.

The latitude correction will be

$C_L = 1.307 \sin 2\phi$ mg/mile N-S

with an uncertainty of 100 meters or .063 miles.

Reduction of Data and Error Estimation

The heading and velocity errors tend to be of a short period, compared to geophysical anomalies, particularly those of large magnitude. Preliminary data screening for "spikes" in the data and appropriate filtering of the Eotvos correction will remove most of these short period variations.

Once these steps have been taken, we may look at the replicate measurements, i.e., the line — Tie-line crossings, to attempt to estimate the data dispersion in our individual measurements.

It is of interest to note the following data as collected by Bedford Institute (Department of Energy, Mines and Resources), covering over 75,000 line miles with 2500 intersections. The data were first examined for bulk errors, i.e. errors so obvious that they represent "spikes" or are otherwise explainable as factually incorrect data.

The deviations between profiles apparently measuring the same value at the same spot were tabulated and a mean absolute deviation of 3.03 mgals was calculated. A % frequency table, which may be used as an estimate of the probable dispersion of errors, was also prepared (Figure 3) (Haworth, Personal Communication, 1973).

A second survey, off the East Coast of North America, shows an average crossing error of 2.7 milligals (Figure 4).

The coincidence of these error values, approximately 3 milligals, probably represents a reasonable estimate of the precision of marine gravity surveys up to 1973. It would be interesting to compile more recent figures, but understandably, contractors and users are reluctant to release such information.

How then can we approach the resolution of these problems in automatic data reduction?

If we have lines of data as in Figure 5 with cross lines intersecting the main lines, the usual products of compilation are profiles and a contour map. The first step in the data reduction chain after spike removal and smoothing of the Eotvos is usually to "rationalize" the errors or misclosures at the line — tie-line intersections. Rationalizing takes many forms but the general intent is minimize these misclosures and in fact to arrive at a zero misclosure so that automatic mapping may proceed. Several types of assumptions are made by different processing groups:

1. The misclosures are due to instrument drift which is not random but a function of time or distance.

2. The misclosures are due to navigation errors which take place only at the inter-

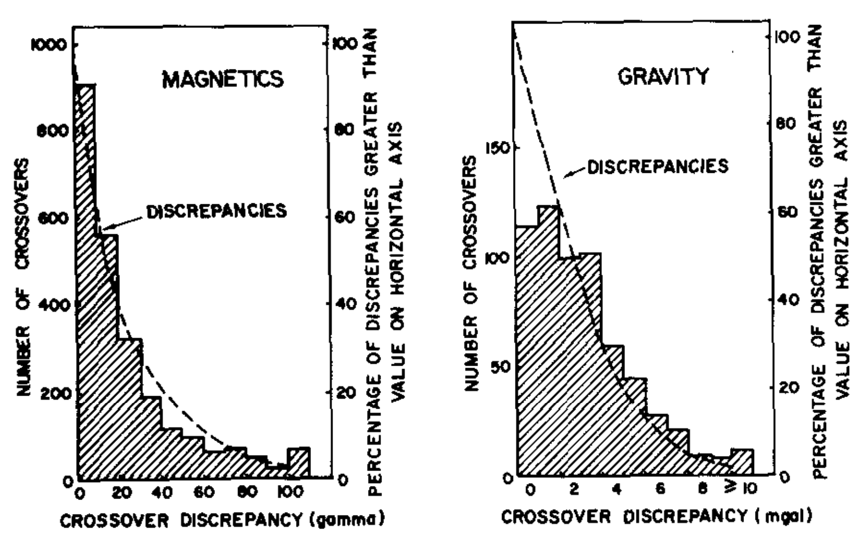

Fig. 3.

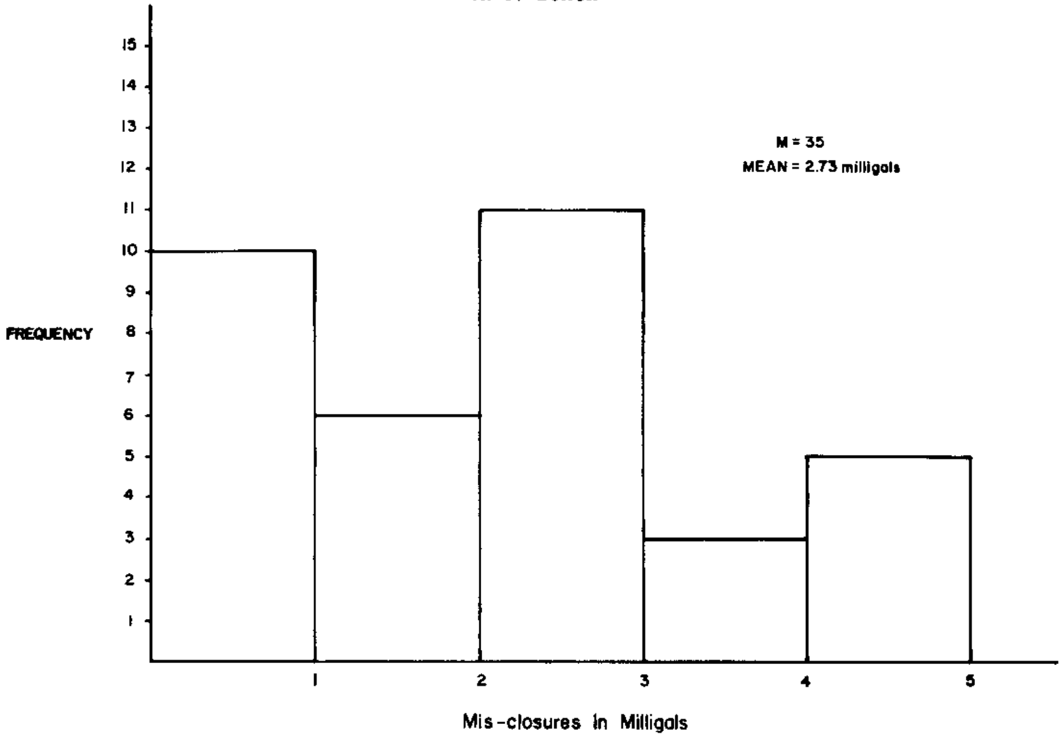
M = 35
MEAN = 2.73 milligals
FREQUENCY
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
Mis-closures in Milligals

Fig. 4.

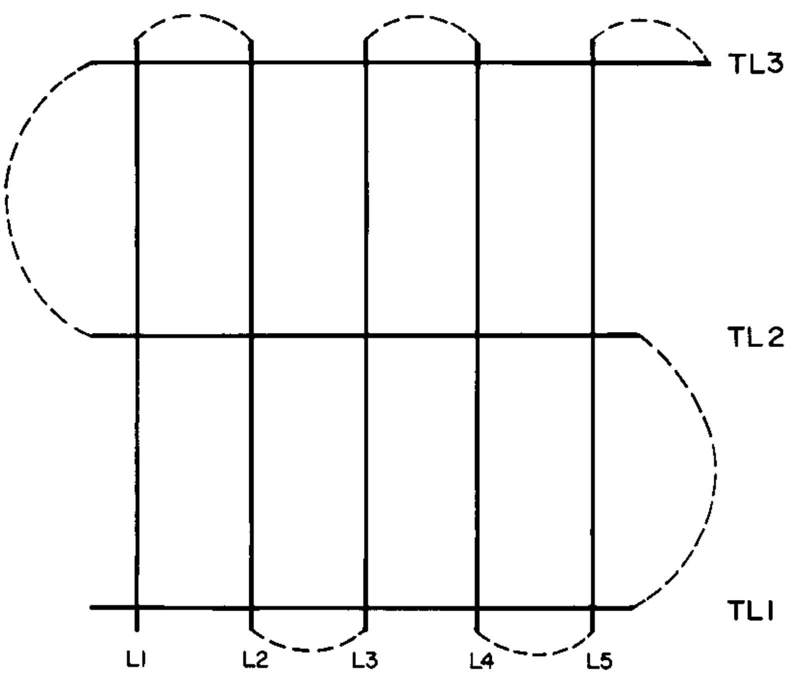
TL3
TL2
TL1
L1
L2
L3
L4
L5

Fig. 5.

section, or within a short distance from the intersection.

3. The misclosures are due to a combination of these events which again are a function of time and may be redistributed over the entire survey.

Assumption 1., which may be true for land data is not true in marine data since the position, heading and velocity of replicate measurements are not identical and are in fact the largest source of error. If this method is used, care should be taken to ensure that the lines over which this error is distributed are continuous in time. If the survey has been broken up into several phases in time this assumption of continuous corrections is completely invalid and attempts to correct for drift by defining an "error surface" or by linear interpolation of error, may result in zero misclosures, but the technique has also added a significant error term to all measurements. This error is very subtle since it is usually continuous between readings and therefore cannot be distinguished from the potential field being measured.

Assumption 2., is obviously false and is only a convenience to arrive at zero closings. Adjusting limited segments of the lines in space may make mapping convenient but does not in any way improve the confidence in the remaining data or make it "more precise". However, in calculating these misclosures, which are in fact replicate measurements in the statistical sense, an estimate of the confidence of the entire survey may be calculated. For example, if a gravity survey has an instrument precision of .05 mg., but the misclosures after smoothing Eotvos corrections average 2 mg. with a standard deviation of .5 mg., then the value of each measurement may be taken with 95% probability (2 S.D.) as ± 3 mg. This most useful measurement encompasses all errors. By then defining the mapping parameters to reflect this probability, e.g. using a contour interval at least 3 mg., the chance of making a wrong decision based on the observed data is minimized.

It may be noted that profiles represent a different case since we have removed one dimension and therefore a large portion of the uncertainty due to errors in spatial position. Therefore, depth estimates from profiles will tend to be more accurate than the positioning of these estimates.

Assumption 3. is most dangerous. The usual processing technique which uses this assumption is to calculate a trend surface of errors as measured by the mis-ties and apply these errors to the entire data set. It is easy to see that no extra precision is added to the individual datum by this technique and in fact, precision is lost by introducing a systematic error term which, since it is continuous, is again indistinguishable from the potential field. The probability of making a wrong decision is increased but is not measureable.

What appears to be the best solution then is to prepare a semi-automatic data analysis chain which allows the interpreter maximum control over the processing parameters. Step 1. would be to profile the observed data, the Eotvos correction and the final corrected gravity. After examination of the short period variations, a "De-Spiking" operation using second difference calculations is carried out to remove single station anomalies. Then a smoothing operator is applied to the Eotvos corrections to attempt to correct for short period errors in heading, location and velocity. At this point, Line — Tie-line misclosures are calculated and a precision estimate made for the survey. It is possible, by close observation by an experienced observer to identify some approximation to instrument drift by noting systematic changes in misclosures.

The final contour interval should reflect the precision estimates made on the initial Tie-line analysis, that is, the contour interval should be no less than the expected error.

Aeromagnetic Data

In Aeromagnetic data collection, the positioning problems are usually not as severe as marine gravity, except in Northern or Jungle areas or areas completely over water, where severe problems do exist. Also another parameter, the diurnal variation in the earth's magnetic field is introduced. This parameter may be estimated by using a fixed station located in the survey area but, except in very localized surveys, this monitor record may vary in phase and amplitude with the conditions measured by the aircraft. Several techniques are available for removing the diurnal variation and the best seems to be to fly a well controlled grid which allows the temporal variation in the field to be measured from line to line or tie-line to tie-line (see Figure 5). The "spikes" (i.e. the rapidly varying component of the diurnal) are first removed and then a smooth curve is fitted to the remaining values of tie-line misclosures (i.e. the slowly varying component of the diurnal). By subtracting these values from the observed data, a good approximation of the diurnal variation may be removed if the survey flight pattern has been planned and executed with this procedure in mind. The residual errors then represent the survey precision and should be used as described in the preceeding marine gravity section to establish the mapping parameters.

Conclusion

In summary, the automatic processing of potential field data should consider the following points:

1. Individual marine gravity, magnetic and aeromagnetic data contain certain random errors due to positioning, velocity, heading and instrument errors as well as serially correllated errors due to diurnal variations, instrument drift and possibly due to systematic errors in positioning. The magnitude of these errors often falls in the range of the anomaly magnitudes under study although they may vary in frequency.

2. The lack of replicate measurements except at tie-line crossings does not allow for the complete, statistical analysis of these errors in normal surveys.

3. Tie-line analysis and misclosure removal does not improve the precision of an individual datum but does indicate the magnitude and dispersion of the errors and may allow an approximation of systematic variations to be calculated.

4. Contour intervals should be chosen on the statistics gained from tie-line analysis or, if insufficient tie-lines are available, chosen on the calculus of variations using estimated velocity, heading and location errors.

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