GRAVITY GRADIOMETRY — A REBIRTH†

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ABSTRACT

Although the Eotvos Torsion Balance has fallen into disuse in geophysical exploration due to its cumbersomeness, recent developments in gravity gradient instrumentation can reasonably be expected to reverse this trend and open new vistas. Gravity gradiometers show promise of supplementing or perhaps supplanting the gravimeter for the measurement of the earth's gravity on a local, regional, or even global scale. Gradiometers are being developed which can be incorporated into moving-base systems capable of gravity surveys over large areas with minimum expenditure of time. Gravity gradients measured with these systems find important application in geophysical exploration and geodetic surveying.

INTRODUCTION

The Eotvos Torsion Balance, introduced in the early 1920s into North America as a tool for geophysical prospecting, measures the local distortion of the gravity field rather than its intensity. This "gradiometer", or instrument for the measurement of the spatial derivatives or gradients of earth's gravity field soon proved rather impractical for field use, especially in poor terrain, because of its cumbersomeness and the lengthy observation sessions. Its sensitivity to even small pebbles in the surrounding area made it necessary to rake the area clean for several meters. It is no wonder that the gravimeter soon replaced it as an exploration tool.

The desire to perform gravity surveys over large areas containing terrain which might not be accessible to vehicles, and the desire that these surveys be accomplished with a minimum expenditure of time and funds led to the investigation of several schemes for airborne gravimetry. Since the gravimeter measures an acceleration it cannot distinguish the attraction due to gravity from vertical accelerations of the carrying vehicle. Further, gravity measurements obtained by a "moving base" gravimeter are subject to serious errors due to uncertainties in the position and velocity of the vehicle. The desire for a suitable platform has led to the suggestion that a rigid airship, i.e., a dirigible, be used as the carrier. This is analogous to submarine gravimetry in the quest for a suitable stable platform.

A gravity gradiometer, on the other hand, does not measure the acceleration due to gravity, but rather the gradients of that acceleration. Thus, a well made gradiometer will not sense the accelerations of the carrier vehicle. Further, the effects of uncertainties of position and velocity are diminished in the gradiometer system. It is, therefore, less necessary to provide the slow moving, highly stabilized carrier so necessary for moving base gravimetry.

GRADIOMETRIC EXPLORATION

For the purpose of defining the gradients of the gravity vector, it is appropriate to establish a cartesian co-ordinate system at the observer's position (Figure 1). The z axis coincides with the direction of earth's center of mass and is defined as positive outward. The x axis is directed northward

†Manuscript received by the Editor September 8, 1975.

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and the y axis eastward, completing a left handed orthogonal triad. In this co-ordinate frame, the gravity gradients at an observation site comprise a tensor

$$\begin{matrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{matrix}$$

Fig. 1. Gradient Coordinate System.

Of the nine components of this tensor, only five are independent in that the matrix is symmetric, i.e., $G_{yx} = G_{xy}$, etc., and Laplace's Equation relates the elements of the diagonal, i.e., $G_{zz} = -(G_{xx} + G_{yy})$.

In the torsion balance era of gravity gradiometry, the measured quantities were the gradients, G_{xz} and G_{yz}, and the curvature $\Delta G = G_{yy} - G_{xx}$ (Nettleton, 1940). Figure 2 illustrates the characteristic gradient, curvature, and gravity signature of a sphere, which salt domes approximate in form. Figure 3 illustrates the signatures of a horizontal cylinder, a form approximating many geologic structures. It is interesting to note that, while the gravity and gradient signatures are quite similar, the curvature is quite distinct for the two features.

It must be emphasized that interpretation of gradiometry data in terms of a definite subsurface mass distribution is by no means unique. Like the gravimetric survey, additional external controls are required, such as seismographic surveys.

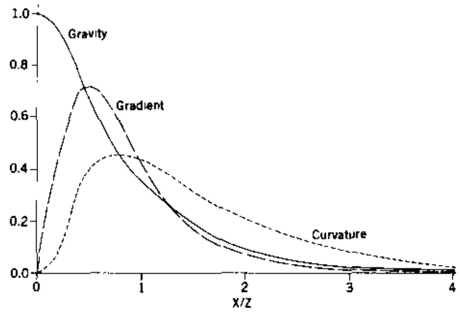

Fig. 2. Gravity, Gradient, and Curvature of a Sphere.

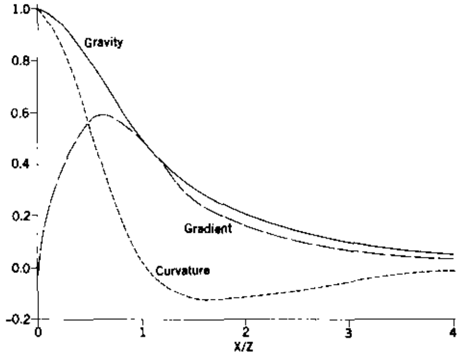

Fig. 3. Gravity, Gradient, and Curvature of a Horizontal Cylinder.

MODERN GRADIOMETRIC INSTRUMENTATION

Since the gravity gradient is defined as the spatial derivative of the gravity vector, the gradiometer must provide a means of determining the slight difference in the acceleration of gravity at two points separated by as little as a few centimeters. The design problem is further complicated by the fact that the gradient tensor is the only quantity to be measured — the instrument cannot be sensitive to temperature gradients, platform jitter or linear accelerations, and electromagnetic or electrostatic influence. In addition, the instrument must have a moving base capability. Presently, three designs known to this author have promise of this capability. The Bell Aerospace Company (Metzger, 1974) has designed an instrument which employs four accelerometers mounted on a rotating table. Two diagonal terms and their cross term of the gradient tensor are derived from a linear combination of the measured accelerations. The Hughes Research Laboratories (Ames, et. al., 1973) has designed an instrument which employs proof masses mounted on pairs of cross arms. Gravity gradients cause the cross arms to torque with respect to each other and the torque is measured by a sensor at the flexure point. Again, two diagonal terms and their cross term are derived from the output of this sensor. Both the Bell Aerospace design and the Hughes Research Laboratories design share the advantage that they are intended to be rotated at an appropriate rate. The gradient signals are modulated at twice the rotational frequency while most noise signals are modulated at the rotational frequency itself. Simple frequency filtering can readily remove the noise from the desired data. The third concept, that of the Charles Stark Draper Laboratory (Trajeser and Johnson, 1974) employs a spherical float supporting a pair of proof masses. A gravity gradient induces a slight rotation of this float, which is sensed by a capacitive pickoff in the float and its housing. Two diagonal terms of the tensor are derived from the output of this instrument. It is rather important to note that this instrument is not rotated, hence, more attention must be paid to isolating the sensor from spurious gradients.

In order to obtain all the components of the gradient tensor, three instruments must be included in a gradiometer package. It is essential that the package be mounted on a platform which isolates it from vibration. The instruments described above are all capable of measuring, when mounted on a suitable platform all components of the gradient tensor with an accuracy of at least 1.0 Eotvos Units (10^{-9}/cm/sec^2/cm) utilizing a 10 second integration time.

EXPLORATION APPLICATION

In order for a reliable gravity map of an area to be developed as expeditiously as possible, it is necessary for the gravity mapping system to obtain a large amount of good data in a minimum of time. A helicopter or other aircraft is the obvious choice for the carrying vehicle, especially if the terrain is rather inaccessible to ground transportation. The position of the vehicle must be known at each recorded observation and the data must be as noise free as possible.

The state-of-the-art in high performance inertial navigation systems is such that a principal source of error is the horizontal gravity anomalies and the associated deflections of the vertical at the vehicle location. These deflections propagate as errors such that the estimated velocity error after four hours flight at 30,000 feet altitude and 540 knots is nearly 75cm/sec (Metzger and Jircitano, 1974). An obvious solution is to introduce these horizontal gravity anomalies into the inertial navigation loop and use them to improve the system. The horizontal gravity anomalies are themselves accelerations and are corrupting the accelerometers in the inertial navigation system. However, horizontal gravity anomalies are integrals of the gravity gradients. Thus, a gravity gradiometer can provide real time information on the gravity field for compensation in the inertial navigation loop. To minimize the problems of bias and white noise in the gradiometer data, a Kalman filter is introduced into the gradiometer/inertial navigation system, (Figure 4). The system outputs accurate position and velocity information and filtered gradiometer data. Since all nine elements of the gradient tensor would be available from the system, any characteristic of the gravity field preferred by the exploration geophysicist can be derived.

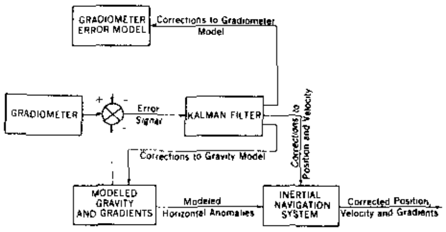

Fig. 4. Gradiometer/Inertial Navigation System.

The gradiometer/inertial navigation system described above, using a gradiometer of 1.0 Eotvos Units accuracy, can map horizontal gravity anomalies to an accuracy of about four milligals and vertical gravity anomalies to an accuracy of less than one milligal (Metzger and Jircitano, 1974).

CONCLUSIONS

The new generation of gravity gradiometers, as accurate as the torsion balance yet suitable for moving base operations, can cause a rebirth of gradiometry in geodetic and geophysical applications. When their output is introduced into inertial navigation systems to compensate for the previously unmeasured horizontal gravity anomalies, an efficient gravity mapping system results. More definitive exploration techniques can then be applied in the areas that the gravity mapping indicates as worthwhile.

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