INTERPRETATION OF MAGNETIC ANOMALIES OF A TWO-DIMENSIONAL FAULT BY FOURIER INTEGRAL†

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ABSTRACT

The Fourier transform of the theoretical horizontal and vertical magnetic anomalies of a two dimensional fault with an inclined face are obtained. The amplitude and phase spectra are then plotted in the frequency domain. It is observed that the amplitude spectrum is the same for both horizontal and vertical magnetic anomalies while the phase spectrum differs by $\pi/2$. The parameters, namely the depth to the top and bottom surface of the fault, the inclination of the face and the susceptibility contrast are determined from the spectra.

LIST OF SYMBOLS

- Z — Vertical magnetic anomaly.
- H — Horizontal magnetic anomaly.
- d — Depth to the top surface of the fault.
- 2h — Vertical throw of the fault.
- k — Susceptibility contrast (neglecting the demagnetising effect).
- i — Inclination or dip of the fault plane.
- β — Angle of strike of the fault from the magnetic north over west.
- H_o, Z_o — Horizontal and vertical components of the earth's magnetic field respectively.
- ω — Angular frequency in radians/km.
- $F_Z(\omega)$, $F_H(\omega)$ — Spectra of vertical and horizontal magnetic anomalies respectively.
- $R(\omega)$, $X(\omega)$ — Real and imaginery part of amplitude spectrum respectively.
- $\phi_Z(\omega)$, $\phi_H(\omega)$ — Phase spectra of horizontal and vertical magnetic anomalies respectively.
- $A(\omega)$ — Amplitude spectrum of magnetic anomalies.
- x — Horizontal distance on earth's surface from the point of origin.

INTRODUCTION

The purpose of this paper is to show the use of the Fourier integral in interpreting magnetic anomalies of two dimensional bodies. The application of the Fourier integral in the interpretation of gravity and magnetic anomalies was done earlier by various investigators such as Bhattacharya (1966) Dean (1958), Odegard and Berg (1965) and many others. Bhattacharya (1966) applied this technique to determine the continuous spectrum of the total magnetic field anomaly due to a rectangular prismatic body. Odegard and Berg (1965) applied the Fourier integral for gravity interpretation and commented that the results of their investigation had been very encouraging.

In the present investigation the theoretical magnetic anomalies of a two dimensional fault with inclined face are considered. The exact Fourier integral of these anomalies are obtained. The amplitude and phase spectra for different nu

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merical values of the depth, throw and dip of the fault face are plotted. It is also shown that these parameters and the susceptibility contrast can be determined from the amplitude and phase spectra if the components of the earth's magnetic field are known.

THEORETICAL ANALYSIS

The vertical and horizontal magnetic anomalies of an inclined fault are given in standard texts (Heiland, 1951). The symbols used are expressed in the 'List of symbols' and also are shown in Figure 1.

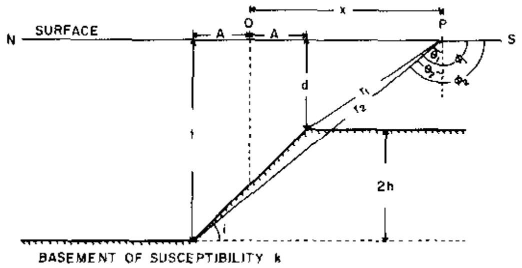

Fig. 1. Cross-section of the fault perpendicular to the strike.

The expressions are given by

$$Z = 2k \operatorname{Sin} i \, [H_o \operatorname{Sin} \beta \, \{\operatorname{Sin} i . \ln (r_2/r_1) + \operatorname{Cos} i \, (\phi_2 - \phi_1)\} - Z_o \, \{\operatorname{Sin} i . (\phi_2 - \phi_1) - \operatorname{Cos} i . \ln (r_2/r_1)\} \,] \quad \ldots(1)$$

$$H = 2k \operatorname{Sin} i \, [H_o \operatorname{Sin} \beta \, \{\operatorname{Sin} i . (\phi_2 - \phi_1) - \operatorname{Cos} i . \ln (r_2/r_1)\} + Z_o \, \{\operatorname{Sin} i . \ln(r_2/r_1) + \operatorname{Cos} i . (\phi_2 - \phi_1)\} \,] \quad \ldots(2)$$

where,

ln stands for natural logarithm,

The Fourier transform of the terms $\ln(r_2/r_1)$ and $(\phi_2 - \phi_1)$ i.e. $(\Theta_2 - \Theta_1)$ are shown in the Appendix-I. The Fourier transform of Z and H are obtained as

$$F_Z(\omega) = [R(\omega)]_Z + j \, [X(\omega)]_Z \text{ and } F_H(\omega) = [R(\omega)]_H + j[X(\omega)]_H$$

where,

$$[R(\omega)]_Z = 2k \operatorname{Sin} i .(\pi/\omega.) \exp. (-\omega d) . [H_o \operatorname{Sin} \beta \, \{\operatorname{Sin}(i+\omega A) - \exp (-2\omega h) . \operatorname{Sin} (i-\omega A)\} + Z_o \, \{\operatorname{Cos} (i + \omega A) - \exp (-2\omega h) . \operatorname{Cos} (i-\omega A)\}] \quad \ldots(3)$$

$$[X(\omega)]_Z = 2k \operatorname{Sin} i .(\pi/\omega) . \exp (-\omega d) . \, [H_o \operatorname{Sin} \beta \, \{\operatorname{Cos}(i+\omega A) - \exp (-2\omega h) . \operatorname{Cos} (i-\omega A)\} - Z_o \, \{\operatorname{Sin} (i + \omega A) - \exp (-2\omega h) . \operatorname{Sin} (i-\omega A)\}] \quad \ldots(4)$$

$$[R(\omega)]_H = -[X(\omega)]_Z \text{ and } [X(\omega)]_H = [R(\omega)]_Z \quad \ldots(5)$$

The amplitude spectrum will, therefore, be the same for both cases and is given by

$$A (\omega) = [\, \{R(\omega)\}^2 + \{X(\omega)\}^2 \,]^{1/2}$$

which after substituting the values of $R(\omega)$ and $X(\omega)$ from equations (3) and (4) and subsequent simplification is expressed as

$$A(\omega) = 2k \operatorname{Sin} i . \, (\pi/\omega) . \exp (-\omega d) . \, [1+\exp (-4\omega h) - 2\exp(-2\omega h) \operatorname{Cos} (2\omega A)]^{1/2} . \, [H_o^2 \operatorname{Sin}^2 \beta + Z_o^2]^{1/2} \quad \ldots(6)$$

where the term $[H_o^2 \operatorname{Sin}^2 \beta + Z_o^2]^{1/2}$ is the component of the total field in the plane perpendicular to the strike of the fault.

The phase spectra are expressed as,

$$\tan [\phi_Z(\omega)] = \frac{[R(\omega)]_Z}{[X(\omega)]_Z} \text{ and } \tan [\phi_H(\omega)] = \frac{[R(\omega)]_H}{[X(\omega)]_H} \quad \ldots(7)$$

It can be shown from (7) with the help of (5) that $\phi_Z(\omega)$ *and* $\phi_H(\omega)$ *have a phase* difference of $\pi/2$.

$$\tan [\phi_Z(\omega)] = [MH_o \operatorname{Sin} \beta - NZ_o] \, / \, [NH_o \operatorname{Sin} \beta + MZ_o] \quad \ldots(8)$$

where,

$$M = \operatorname{Cos} (\eta) - \exp (-2\omega h) . \operatorname{Cos} (\psi)$$

$$N = \operatorname{Sin} (\eta) - \exp (-2\omega h) . \operatorname{Sin} (\psi)$$

$$\eta = i + \omega A \text{ and } \psi = i - \omega A$$

As ω tends to zero the value of $\tan [\phi_Z(\omega)]$ can be obtained from (8) by using L'Hopitals rule. This gives

$$\operatorname*{Lt}_{\omega=0} \tan [\phi_Z(\omega)] = \frac{-Z_o}{H_o \operatorname{Sin} \beta} = \tan [\phi_Z(0)]$$

and $\ldots(9)$

$$\operatorname*{Lt}_{\omega=0} \tan [\phi_H(\omega)] = \frac{H_o \operatorname{Sin}\beta}{Z_o} = \tan [\phi_H(0)]$$

INTERPRETATION OF AMPLITUDE AND PHASE SPECTRA

The expression in the equation (6) shows the amplitude spectrum for both horizontal and vertical magnetic anomalies. A plot of the amplitude spectrum versus the angular frequency, ω, *is shown in Figure* 2. It can also be verified from (6) that,

$$\operatorname*{Lt}_{\omega \to 0} [A(\omega)]^2 = (4 \pi k h T)^2 \quad \ldots(10)$$

where $T = [H_o^2 \operatorname{Sin}^2 \beta + Z_o^2]^{1/2}$

Now let the function $\omega^2 [A(\omega)]^2$ be defined as $f(\omega)$ then

$$f(\omega) = 4\pi^2 k^2 T^2 \operatorname{Sin}^2 i . \exp (-2\omega d) . \, [1 + \exp (-4\omega h) - 2\exp (-2\omega h) . \operatorname{Cos} (2\omega A)] \quad \ldots(11)$$

As ω becomes large $f(\omega)$ can be approximated by $f(\omega) = 4 \pi^2 k^2 T^2 \operatorname{Sin}^2 i . \exp (-2\omega d)$

or $$\ln [f(\omega)] = 2 \ln (2 \pi k T \operatorname{Sin} i) - 2\omega d \quad \ldots(12)$$

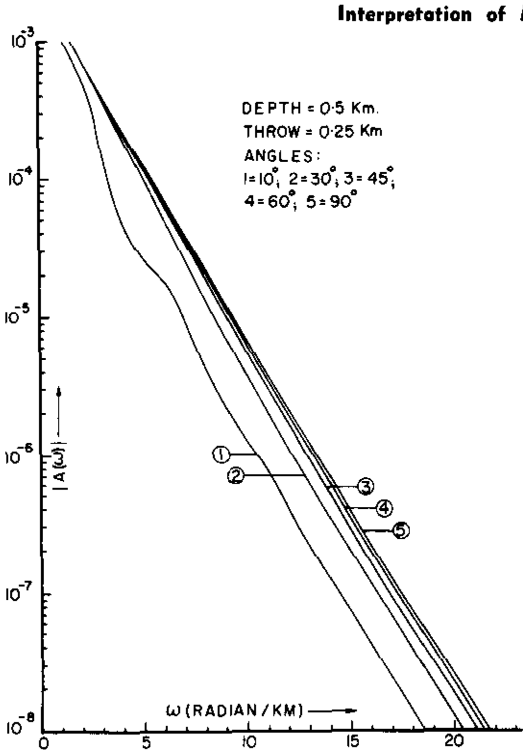

Fig. 2. Amplitude spectra for different angles of inclination of the fault plane

The above expression represents a straight line with a negative slope 2d and an intercept of ln $(2\pi k T \sin i)^2$. For smaller values of ω, however, the function $f(\omega)$ will show the combined nature of the exponential terms but for larger values will contain the contribution of only one exponential term. This behaviour of $f(\omega)$ is shown in the semi-logarithmic plot of Figure 3. It is also evident from (12) that the slope of the straight line part in Fig. 3 gives the value of d which is the depth to the top surface of the fault. The technique of analysing multiple decay spectra (Kaplan, 1965) could not be tried to determine the value of h from $f(\omega)$ because of the term $\cos(2\omega A)$. However, it can be known from the behaviour of the phase spectrum.

By trigonometric simplification the expression for the phase spectrum in (8) can be written as

$$\tan[\phi_Z(\omega)] = \tan[\phi_Z(0) + \phi_Z(x)] \quad \ldots(13)$$

where,

$$\tan[\phi_Z(x)] = \frac{\cos(i+\omega A) - \exp(-2\omega h).\cos(i-\omega A)}{\sin(i+\omega A) - \exp(-2\omega h).\sin(i-\omega A)}$$

$$= \cot[i+\phi(y)] = \tan[\pi/2 - i - \phi(y)] \quad \ldots(14)$$

$$\tan[\phi(y)] = \tan(\omega A).[1 + \exp(-2\omega h)] / [1-\exp(-2\omega h)]$$

$$= \tan(\omega A).\coth(\omega h)$$

For large ω, $\coth(\omega h) = 1$ so

$$\tan[\phi(y)] = \tan(\omega A) \text{ or } \phi(y) = n\pi + \omega A$$

$$n = 0, 1, 2, \ldots$$

The phase spectrum can now be expressed as $\phi_Z(\omega) = (2n+1)\frac{\pi}{2} + \phi_Z(0) - i - \omega A \quad \ldots(15)$

for larger values of ω.

The expression (15) shows that $\phi_Z(\omega)$ versus ω will be a straight line for higher values of ω and the slope of the line will be equal to A i.e. h Cot i. The plot of $\phi_Z(\omega)$ versus ω is shown in Figure 4, The variation of $\phi_Z(\omega)$ in Figure 4, apparently does not show a linear behaviour. This is due to the term $(2n+1).\pi/2$ which has not been taken into consideration. The tangent function has discontinuity at the points $\pm n\pi/2$. Therefore, at the point of discontinuity each segment should be shifted by an amount of π with respect to the previous segment. This plot is shown in Figure 5 where $\phi_Z(\omega)$ versus ω now behaves linearly for higher values of ω. The extrapolation of the linear part intersects the $\phi_Z(\omega)$ axis at $(\pi/2 + \phi_Z(0) - i)$ As $\phi_Z(0)$ can be known independently from the values of the earth's field in an area and also from the plot of $\phi_Z(\omega)$ versus ω, the value of i can be found out from this intercept. Once i is known h can be computed as A, and hence h Cot i, is known from the slope of the linear part of Figure 5. The value of A can also be determined from Figure 4 by noting the consecutive values of ω when $\phi_Z(\omega)$ equals to O. The value of A can then be obtained from the expression $(\omega_2 - \omega_1) A = \pi/2$.

The parameters d, h and i being determined k can be obtained from the intercept of $[A(\omega)]^2$ a plot of which is shown in Figure 6. It should be noted that for all these figures the value of β is taken as $\pi/2$, that is the strike of the fault is in the E-W direction.

Conclusions

It has been shown that where the source of a magnetic anomaly can be approxi-

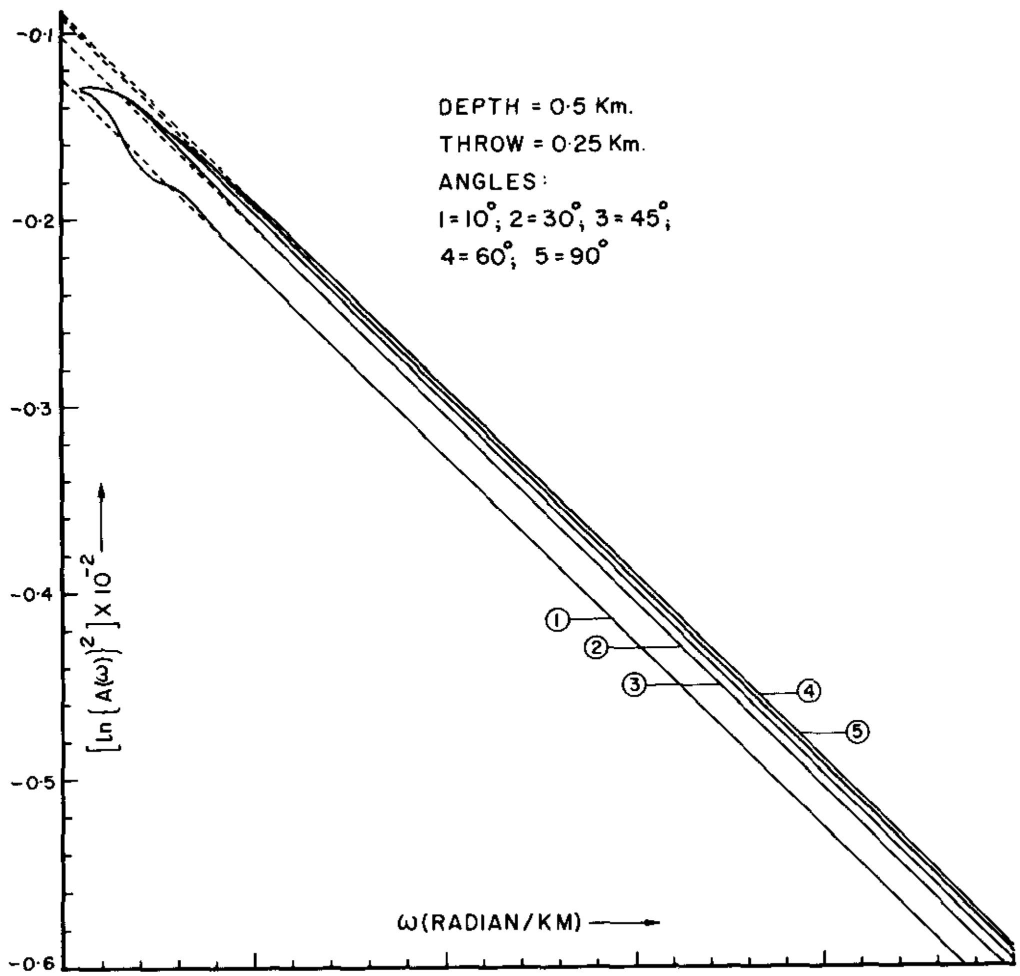

Fig. 3. Modified power spectra for different angles of inclination of the fault plane.

mated by a two dimensional fault, it should be possible to determine the parameters from the amplitude and phase spectra. For a vertical fault (i.e. when i = 90°, A = O), the expressions for the amplitude and phase spectra are

$$A(\omega) = 2\ kT.(\pi/\omega).\ [\exp(-\omega d) - \exp(-\omega d - 2\omega h)]$$

and $$\phi_Z(\omega) = -(Z_o/H_o \ \mathrm{Sin}\ \beta)$$

The parameters d and h can be determined from the plot of $[A(\omega)]$ on a semilogarithmic scale. For higher values of ω ln $[\omega\{A(\omega)\}]$ will be straight line with slope —d. The value of h can then be obtained by using the method of analysing multiple decay spectra (Kaplan, 1955).

An identical process of interpretation can be applied for the magnetic anomalies due to an inclined fault of a bed of finite thickness. Also, this technique can be successfully applied for other two dimensional bodies like dikes and horizontal cylinders. Further, it has been shown by other investigators (e.g. Eby, 1972; Odegard & Berg, 1965) that a similar method works for gravity anomalies due to the assemblages of two or more sources even though the source parameters vary. Sharma & Geldart (1968) have successfully applied

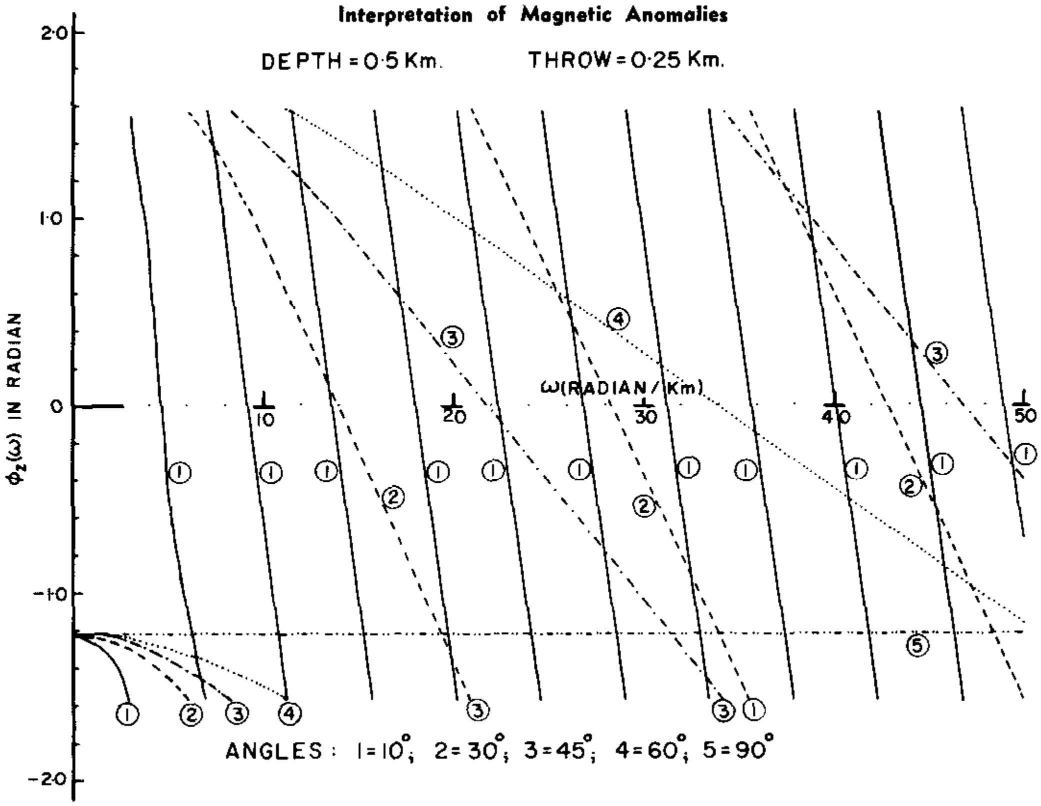

Fig. 4. Phase spectra of the vertical anomalies for different angles of inclination of the fault plane.

similar techniques of frequency analysis to the gravity data over the Logan fault area.

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Appendix — I

Let the Fourier transform of ln (r_2/r_1) be given as $F(\omega)$. Then

$$F(\omega) = 1/2 \int_{-\infty}^{\infty} \ln \{(x+A)^2 + t^2\} \,.\, \exp(-j\omega x)\, dx$$

$$- 1/2 \int_{-\infty}^{\infty} \ln \{(x-A)^2 + d^2\} \,.\, \exp(-j\omega x)\, dx$$

where $j = \sqrt{-1}$ and $\exp(-j\omega x) = e^{-j\omega x}$

Substituting $(x+A) = y$ and $x-A = z$,

$$F(\omega) = 1/2 \exp(j\omega A) \int_{-\infty}^{\infty} \ln(y^2+t^2).\, \exp(-j\omega y)\, dy$$

$$- 1/2 \exp(-j\omega A) \int_{-\infty}^{\infty} \ln(z^2+d^2).\, \exp(-j\omega z)\, dz$$

As $\ln(y^2+t^2)$ and $\ln(z^2+d^2)$ are even functions of y and z respectively, so

$$F(\omega) = \exp(j\omega A) \int_{-\infty}^{\infty} \ln(y^2 + t^2) \cos \omega y\, dy$$

$$- \exp(-j\omega A) \int_{-\infty}^{\infty} \ln(z^2 + x^2) \cos \omega t\, dz$$

$$= \exp(j\omega A).\, [-\frac{\pi}{\omega} \,.\, \exp(-\omega t)] + \exp(-j\omega A)\, [\frac{\pi}{\omega} \exp(-\omega d)]$$

(Erdelyi, A., et al. 1954, p-18 eq. 12)

The real and imaginary parts of $F(\omega)$ are given as

$$\text{Re } F(\omega) = (\pi/\omega).\, \exp.\, (-\omega d).\, [1-\exp(-2\omega h)].\, \cos \omega A$$

and

$$\text{Im } F(\omega) = -j(\pi/\omega).\, \exp(-\omega d).\, [1+ \exp(-2\omega h)].\sin \omega A$$

respectively.

Fourier transform of $(\theta_2 - \theta_1)$

Let the Fourier transform of $(\theta_2 - \theta_1)$ be given as $G(\omega)$ Then using the same logic as in the previous case

$$G(\omega) = \int_{-\infty}^{\infty} \arctan[(x+A)/t].\, \exp(-j\omega x) dx$$

$$- \int_{-\infty}^{\infty} \arctan[(x-A)/d].\, \exp(-j\omega x(dx)$$

$$= \exp(j\omega A) \int_{-\infty}^{\infty} \arctan(y/t).\, \exp(-j\omega y)\, dy$$

$$- \exp(-j\omega A) \int_{-\infty}^{\infty} \arctan(z/d).\, \exp(-j\omega z)\, dz$$

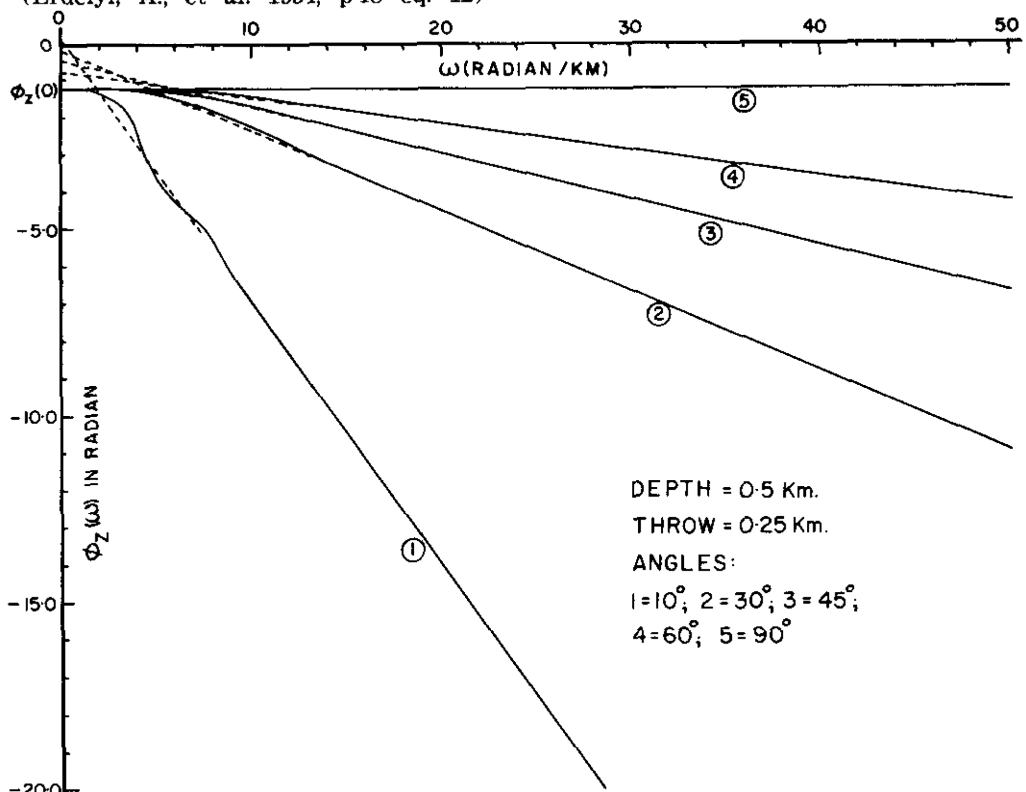

Fig. 5. Phase spectra of the vertical anomalies for different angles of inclinations of the fault plane.

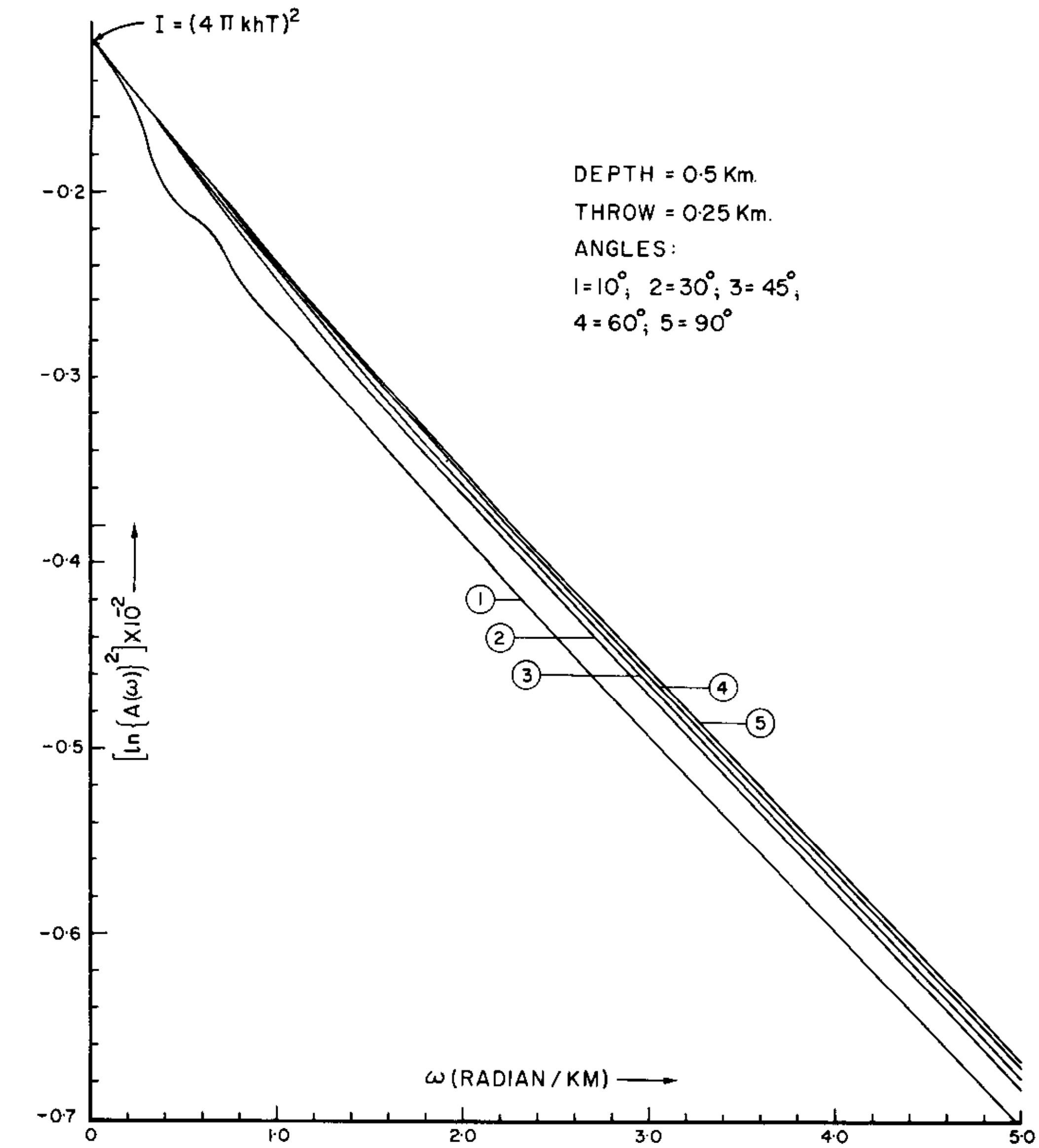

Fig. 6. Power spectra for different angles of inclination of the fault plane.

As both arctan (y/t) and arctan (z/d) are odd functions of y and z respectively

$$G(\omega) = -2j \exp(j\omega A) \int_{-\infty}^{\infty} \arctan(y/t) . \operatorname{Sin} wy \, dy + 2j \exp(-j\omega A) \int_{-\infty}^{\infty} \arctan(z/d) . \operatorname{Sin} \omega z \, dz$$

$$= -2j \exp(j\omega A) . \left[\frac{\pi}{2\omega} \exp(-\omega t)\right] + 2j \exp(-j\omega A) . \left[\frac{\pi}{2\omega} \exp(-\omega d)\right]$$

(Erdelyi, A. et al., 1954, p-87, eq. 3)

The real and imaginary part of $G(\omega)$ are respectively

$$\operatorname{Re} G(\omega) = (\pi/\omega) . \exp(-\omega z) . [1 + \exp(-2\omega h)] . \operatorname{Sin}(\omega A)$$

and $$\operatorname{Im} G(\omega) = j(\pi/\omega) . \exp(-\omega z) . [1 - \exp(-2\omega h)] . \operatorname{Cos}(\omega A)$$