SOME NOMOGRAMS FOR GRAVITY INTERPRETATION OF THE ZERO DEPTH STEP MODEL

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ABSTRACT

In certain situations, for example Precambrian areas, the depth to the top of rock masses is known to be zero. The remaining variables, thickness, density, and attitude can then be determined with somewhat greater confidence than when the depth is also an unknown. Some interpretation procedures are presented here for the interpretation of the zero depth step model with dipping or vertical face. The step model can be used to approximate a linear, steeply dipping contact such as a fault contact or steeply dipping beds.

THE ZERO DEPTH STEP MODEL

Characteristic curves were presented by Grant and West (1965) for the gravity step model. Their curves are designed for a buried step and depend on the measurement of the maximum slope of the curve.

Unfortunately, when the step model is close to the surface or at zero depth, the maximum slope is large or infinite so that it cannot be used to enter their graphs.

Suitable interpretation graphs can be developed if their equations are rewritten and used to generate parameters.

The equation for the sloping step as given by Grant and West is

$$\Delta g(x) = 2G\rho \left[\frac{\Pi L}{2} + (h+L) \tan^{-1} \frac{x - L \cot d}{h + L} - h \tan^{-1} \frac{x}{h} \right.$$
$$+ 1/2 \,(x \sin^2 d + h \sin d \cos d) \ln \frac{(x - L \cot d)^2 + (h+L)^2}{x^2 + h^2}$$
$$\left. - (x \sin d \cos d + h \cos^2 d)(\tan^{-1} \frac{x - L \cot d}{h + L} - \tan^{-1} \frac{x}{h}) \right] \quad (1)$$

and the first derivative is

$$\frac{d\, \Delta g(x)}{d\, x} = G\rho \left[\sin^2 d \ln \frac{(x - L \cot d)^2 + (h+L)^2}{x^2 + h^2} \right.$$
$$\left. -2 \sin d \cos d \,(\tan^{-1} \frac{x - L \cot d}{h + L} - \tan^{-1} \frac{x}{h}) \right] \quad (2)$$

and the second derivative is

$$\frac{d^2 \Delta g(x)}{d\, x^2} = 2\, G\rho L \, \frac{L\, h \cot d - 2xh - xL}{((x - L \cot d)^2 + (h+L)^2)\;(x^2 + h^2)}$$

where h is the depth to top, L is the thickness, x the distance measured from the top edge of the slab, and d is the dip of the face measured from the positive x direction.

If the top is set to zero depth the equations are:

$$\Delta g(x) = 2\, G\rho \left[\frac{\Pi L}{2} + L \tan^{-1} \frac{x - L \cot d}{L} \right.$$
$$+ 1/2\, x \sin^2 d \ln \frac{(x - L \cot d)^2 + L^2}{x^2}$$
$$\left. - x \sin d \cos d \,(\tan^{-1} \frac{x - L \cot d}{L} - \tan^{-1} \frac{x}{0}) \right] \quad (3)$$

and

$$\frac{d\, \Delta g(x)}{dx} = G\rho \left[\sin^2 d \ln \frac{(x - L \cot d)^2 + L^2}{x^2} \right.$$
$$\left. -2 \sin d \cos d \,(\tan^{-1} \frac{x - L \cot d}{L} - \tan^{-1} \frac{x}{0}) \right] \quad (4)$$

where $\tan^{-1} \frac{x}{0} = \frac{\Pi}{2}$ for $+x$

0 for $x = 0$

$\frac{-\Pi}{2}$ for $-x$

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Equations (3) and (4) can be used to derive a set of characteristic curves complementary to those presented by Grant & West. Charts 1 and 2 show such a set.

To use these curves the interpreter should decide whether or not his gravity anomaly should be treated as a zero depth step model. For a zero depth model, the theoretical slope at the contact line is vertical and marks the inflection. Call that point X_0. Another point should be chosen near the "Knee" of the curve where the curvature is greatest. Measure the slope S of the tangent at the chosen point. Find the same tangent point on the other limb of the curve. Call the upper point X_2 and the lower point X_1. Determine g_2 and g_1.

Form the ratios $\frac{g_2-g_1}{(x_2-x_1)S}$ and $\frac{x_1-x_0}{x_2-x_0}$

if $\frac{x_1-x_0}{x_2-x_0}$ is greater than unity,

use its inverse. The dip direction will then be towards x_1 instead of x_2.

Enter chart one with the calculated parameters. Interpolate the dip and X/L values.

The thickness or depth to bottom, L, is $\frac{(x_2-x_1)}{X/L}$.

Enter chart two with the same parameters. Interpolate $\rho L/g$.

The density, ρ, is $\frac{g_2-g_1}{L}\ \frac{\rho L}{g}$.

To check the interpretation, choose another point near the knee of the field curve and repeat. All such points should yield similar results if the model is really a zero depth step model.

If the dip of the face is vertical, charts 1 and 2 can be used in the normal way if desired. However the simplification permits another easy approach.

The interpreter arbitrarily chooses a point, X_2, on the upper limb of the curve with slope S, as previously, and the other point, X_1, with the same slope on the lower limb.

Then points X_4 and X_3 are found which have slope 1/2 S.

Using equation (4) with d = 90°, we have

$$S = GP\ 1n\ \frac{(X_2-X_1)^2 + 4L^2}{(X_2-X_1)^2} \qquad (5)$$

$$1/2\ S = GP\ 1n\ \frac{(X_4-X_3)^2+4L^2}{(X_2-X_1)^2}$$

Dividing and simplifying we obtain

$$L = \frac{X_4-X_3}{2}\left[\left(\frac{X_4-X_3}{X_2-X_1}\right)^2 -2\right]^{1/2} \qquad (6)$$

and if L is substituted back into (5) a value of ρ can be found.

It may help the interpreter using these slope methods to plot the slope or the first difference of the gravity curve to aid in choosing points.

REFERENCES

Grant, F. S. and West, G. F., 1965. Interpretation Theory in Applied Geophysics. New York, McGraw-Hill.

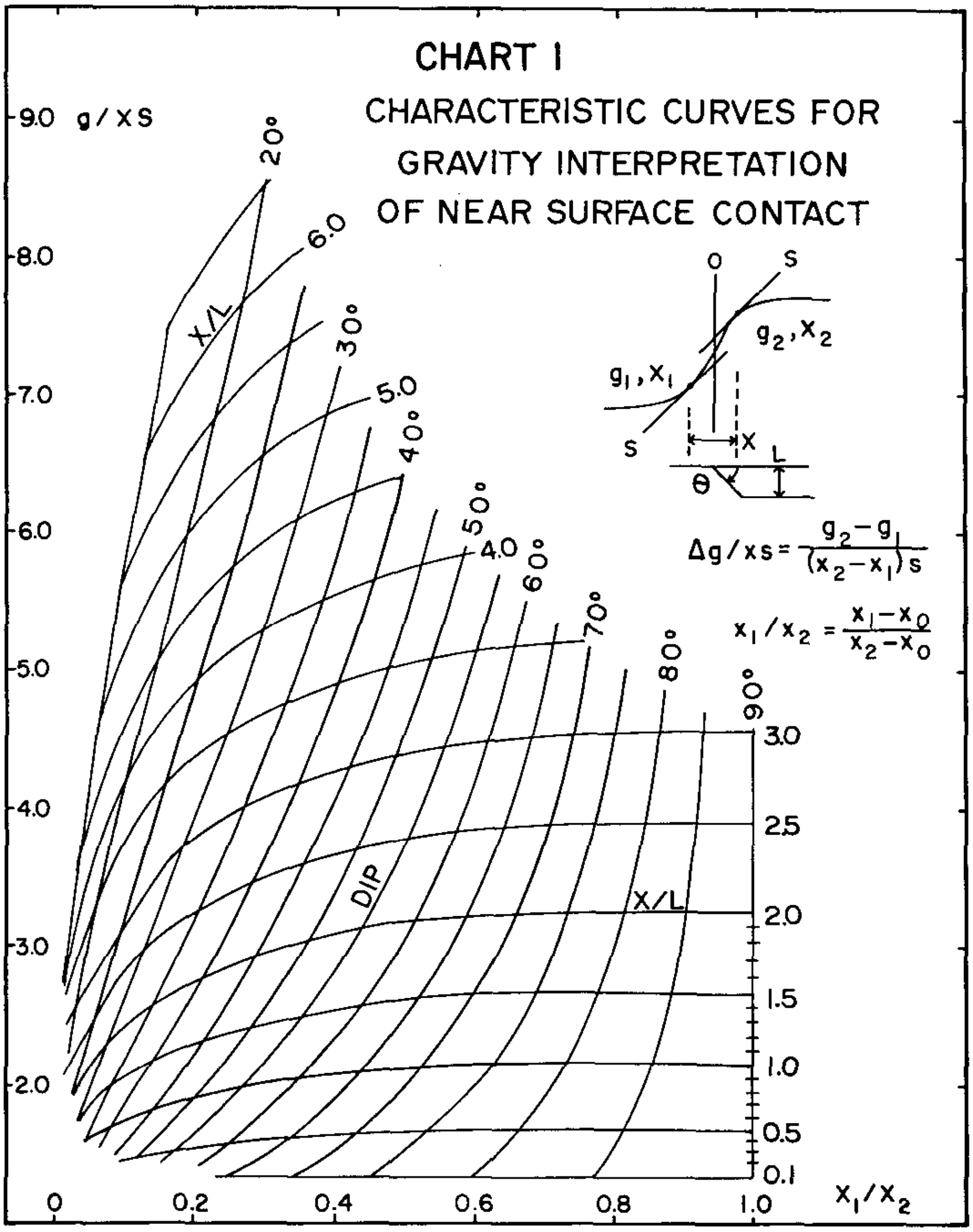
CHART I
CHARACTERISTIC CURVES FOR
GRAVITY INTERPRETATION
OF NEAR SURFACE CONTACT
g/XS
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
20°
30°
40°
50°
60°
70°
80°
90°
6.0
5.0
4.0
X/L
DIP
X/L
3.0
2.5
2.0
1.5
1.0
0.5
0.1
0
0.2
0.4
0.6
0.8
1.0
X1/X2
O
S
g2, X2
g1, X1
S
X
L
θ
Δg/xs = (g2 − g1) / ((x2 − x1) s)
x1/x2 = (x1 − x0) / (x2 − x0)

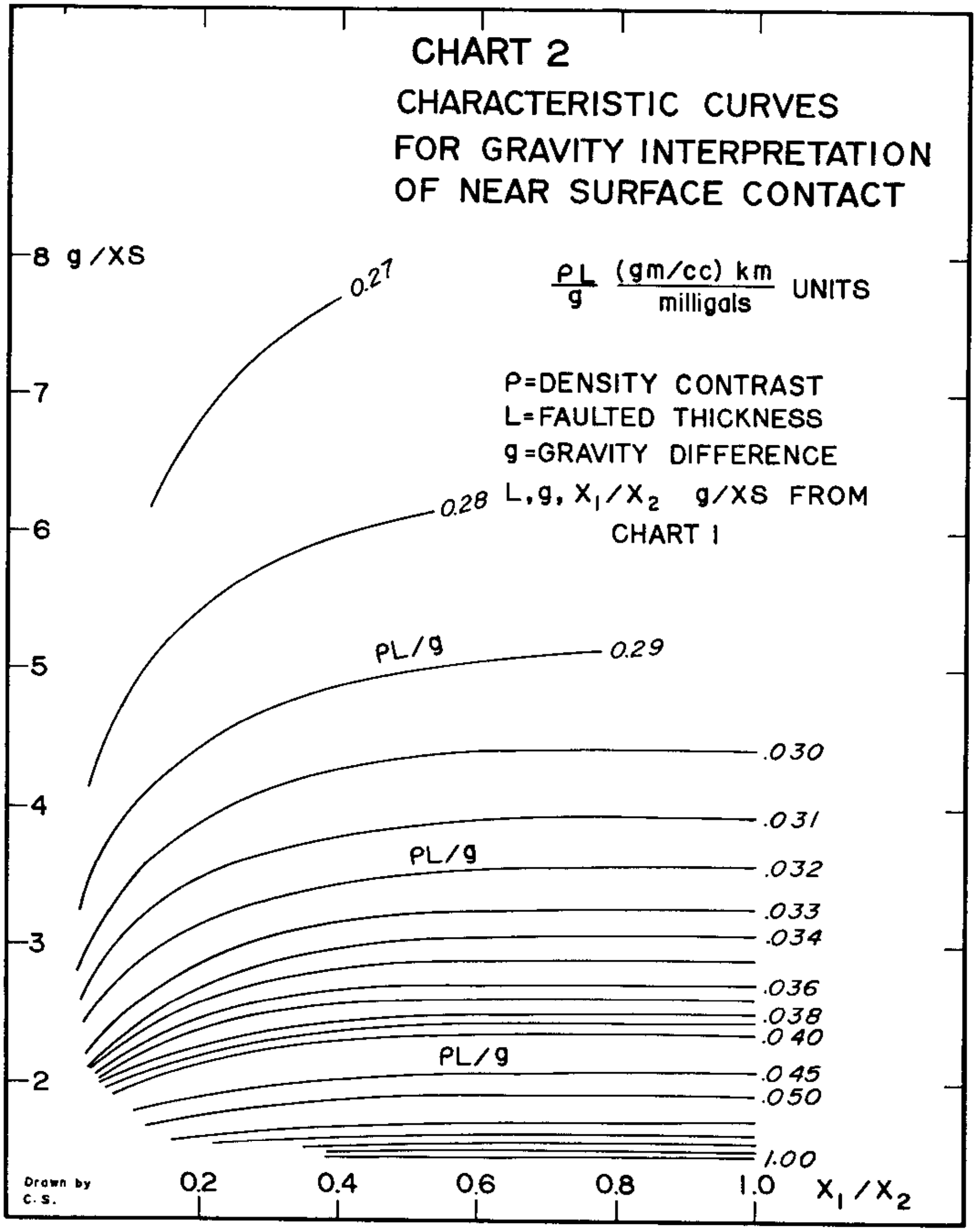
CHART 2
CHARACTERISTIC CURVES
FOR GRAVITY INTERPRETATION
OF NEAR SURFACE CONTACT
ρL/g (gm/cc) km / milligals UNITS
ρ=DENSITY CONTRAST
L=FAULTED THICKNESS
g=GRAVITY DIFFERENCE
L, g, X1/X2 g/XS FROM
CHART 1
g/XS
8
7
6
5
4
3
2
0.27
0.28
0.29
PL/g
.030
.031
.032
.033
.034
.036
.038
.040
.045
.050
1.00
0.2
0.4
0.6
0.8
1.0
X1/X2
Drawn by C. S.