LOW FREQUENCY RECOVERY IN THE INVERSION OF SEISMOGRAMS +

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ABSTRACT

The inversion of seismograms to acoustic impedance logs would at first appear to be a matter of applying the transformation

$$Z_2 = Z_1 \frac{(1 + R_1)}{(1 - R_1)}$$

A study of the low amplitude ($|R| < 0.3$) approximation

$$R(t) = \frac{1}{2}\frac{d}{dt} \quad (\log Z(t)) \text{(Peterson et al. 1955)}$$

reveals some of the difficulties which may be encountered in a practical application of the transform. For example, addition in 'reflection space' becomes multiplication in 'acoustic impedance space'. Further, nonlinearity of the transform can lead to erroneous results when frequency filtering is attempted on the acoustic impedances. The fact that seismograms usually lack signal energy in the 0 - 10 Hz. band leads us to look for another source to fill this 'hole'. RMS stacking velocities are, in general, not adequate for the required purpose and resort is made to sonic logs of wells on or near the seismogram under study. A technique to combine low frequency well information with high frequency seismic information consists of:

(i) A manual interpretation of major events clearly identifiable on the seismic and well-log information.

(ii) A cross correlation to 'fine-tune' the interpretation.

(iii) A 'stretching' algorithm to produce pseudo well logs at every seismic trace position. This algorithm preserves gradients in the original log while stretching (or compressing) the flat (constant interval velocity) portions of the log.

(iv) Combining the low frequency portions of these pseudo logs with a scaled version of the seismic trace.

Although the process appears to be a simple mapping of the seismic trace to an acoustic impedance 'trace', synthetic and real examples show the value of the transform as a practical aid in interpretation.

INTRODUCTION

The inversion of seismograms to pseudo-acoustic impedance logs was first discussed as early as 1963 by Kunetz. More recent contributions are from Delas et al (1970), Lavergne (1975), Lavergne and Willm (1977), and Beitzel et al (1977).

Most authors have recognized the fact that seismic data has very little useful energy in the frequency band of apprxoimately 0-10 Hz., and that some attempt must be made to fill this spectral hole.

First of all we will cover the definition and some of the more important properties of the transform from reflection coefficients to acoustic impedance. Then we will look at some possible candidates for the job of filling the spectral hole and show that the best one is an acoustic log of a nearby well. We will show a way to modify an acoustic log so that we essentially build a low frequency version of the seismogram. Finally, using the properties of the transform, we will combine the low frequency traces and the seismogram to obtain pseudo acoustic-impedance logs.

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THE VELCITY TRANSFORM

The usual equation defining the reflection coefficient R at the boundary between two media of different acoustic impedances $\rho_1 V_1$ and $\rho_2 V_2$ is

$$R = \frac{\rho_2 V_2 - \rho_1 V_1}{\rho_2 V_2 + \rho_1 V_1} \qquad (1)$$

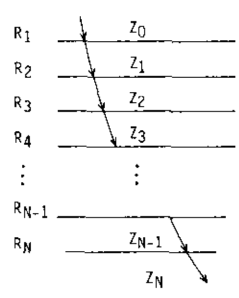
or re-writing this in terms of the acoustic impedance

$$\rho_2 V_2 = \rho_1 V_1 \frac{1 + R}{1 - R} \qquad (2)$$

We note that this formulation of reflection coefficients and acoustic impedance is discrete. Modern processing of seismic data is carried out digitally on discrete time — sampled series. However, the development of deconvolution and filtering techniques is based upon the study of continuous time signals. So, we may gain insight into the inversion of seismic signals if we can derive a continuous expression for the acoustic impedance series.

Let us examine a N + 1 layer model with N reflection coefficients R_i ($i = 1, 2, \ldots N$) and N + 1 acoustic impedances Z_k ($k = 0, 1, 2, \ldots N$)

Let the two way travel time in each layer be λ, we can always arrange that the layers have the same thickness by simply constructing more and more intervening layers with zero reflection coefficients. The final number of layers will always be finite.



At each interface we have

$$R_i = \frac{Z_i - Z_{i-1}}{Z_i + Z_{i-1}} \qquad i = 1, 2, \ldots N$$

$$\text{or } Z_i = Z_{i-1} \frac{(1 + R_i)}{(1 - R_i)} \qquad i = 1, 2, \ldots N$$

$$\text{i.e. } Z_i = Z_0 \prod_{k=1}^{i} \frac{(1 + R_k)}{(1 - R_k)}$$

Taking logarithms we have

$$\log \frac{Z_i}{Z_0} = \sum_{k=1}^{i} \log \frac{(1 + R_k)}{(1 - R_k)} \qquad i = 1, 2, \ldots N$$

We will approximate $\log \frac{1 + R_k}{1 - R_k}$ by $2R_k$.

This is good to second order in R_k

$$\log \frac{1 + x}{1 - x} = 2\left(x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots\right) \quad (x^2 < 1)$$

$$\text{Thus } \log \frac{Z_i}{Z_0} = 2 \sum_{k=1}^{i} R_k \qquad i = 1, 2, \ldots N \quad (3)$$

The error will be less than $2i\left[\frac{R_{max}^3}{3} + \frac{R_{max}^5}{5} + \ldots\right]$

where $R_{max} = \max \{R_1, R_2, R_3, \ldots R_N\}$

An example of the error in a real case is shown in Figure 1a. This is a graph of the percentage error in the acoustic impedances Z_i as computed from a given set of reflection coefficients by the two formulae — the first (correct) formula using the expression $\log \frac{1 + R_k}{1 - R_k}$ the second (aproximate) formula using the expression $2R_k$. The reflection coefficients R are those derived from an actual sonic log which is fully displayed in Figure 2. The largest error is of the order of 1% while the average error is of the order of 0.5%. A point worth noting is that the error does not increase with time, a fact we can attribute to the random appearance of the reflection coefficient sequence; i.e. it has zero algebraic mean.

Now introducting λ, we have

$$\log \frac{Z_i}{Z_0} = \frac{2}{\lambda} \sum_{k=1}^{i} R_k \cdot \lambda$$

If now, we assume that R_k, is a sample of a constant function $(-\infty < t' < \infty)$, $r_k(t') = R_k$, then over the interval $[(k - 1)\lambda, k\lambda]$, it is easy to prove that

$$\log \frac{Z_i}{Z_0} = \frac{2}{\lambda} \sum_{k=1}^{i} \int_{(k-1)\lambda}^{k\lambda} r_k(t')dt'$$

and if we now define a piecewise - continuous function of time, $r(t')$ as the union of all the $r_k(t')$, each on its own interval, we can combine all the separate integrals.

Thus in the interval [k − 1) λ, kλ] we have $r(t') \equiv r_k(t')$ — this is true for all k = 1, 2, 3, . . ., i

Hence, $$\log \frac{Z_1}{Z_0} = \frac{2}{\lambda} \sum_{k=1}^{i} \int_{(k-1)\lambda}^{k\lambda} r(t')dt'$$

and finally $$\log \frac{Z_1}{Z_0} = \frac{2}{\lambda} \int_{0}^{i\lambda} r(t')dt'$$

If we now introduce t = iλ, we can write an equation involving a continuous function Z(t)

Namely $$\log \frac{Z(t)}{Z_0} = \frac{2}{\lambda} \int_{0}^{t} r(t')dt' \qquad (4)$$

The λ which appears may be thought of as the sampling rate where seismic data is concerned. Or as it appears here, it may be disregarded as in arbitrary scaling factor.

Hence we will write down the continuous analogs of equations (1) and (2)

$$r(t) = \frac{1}{2} \frac{d}{dt} \left\{ \log z(t) \right\} \qquad (5)$$

$$z(t) = z_0 \exp \left\{ \int_{0}^{t} r(t')dt' \right\} \qquad (6)$$

where the lower case letter r, z are used to emphasize the fact that these are continuous functions of time. We define z as the continuous analog of acoustic impedance ρV.

From the curve of actual R versus the logarithmic approximation, Figure 1b, we can see that the error in the range $0 \leq |R| \leq 0.3$ is extremely small and hence our equations (5) and (6) should be accurate for the majority of seismic data.

We'll now define a function of r(t) which we'll call the Velocity Transform of r(t)

$$V(r) = \exp \left\{ \int_{0}^{t} r(t')dt' \right\} \qquad (7)$$

Such a transform has certain properties. Below we show two of them:

$$V(p + q) = V(p) \cdot V(q) \qquad (8)$$

$$V(a \times p) = e^{a} \cdot V(p) \qquad (9)$$

where p (t) and q (t) are functions of time and 'a' is a constant.

The first property is the most interesting one to us in this discussion.

Suppose, for example, we split a trace into low and high frequency components, then although we add the reflection coefficient to get back to the original trace we see that we must multiply the velocity transforms of the two components to get back the full frequency velocity function.

From properties (8) and (9) above it is obvious that the transform is non-linear. Let us examine just how serious an error this might cause.

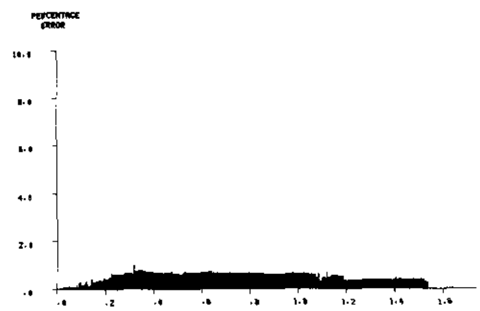


Fig. 1a. Percentage error using approximate formula for acoustic impedances.

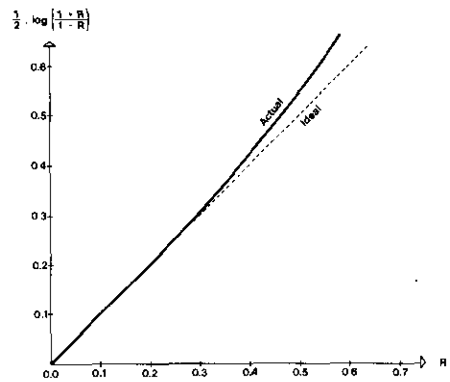


Fig. 1b. Curve of reflection coefficient, R, versus its logarithmic approximation.

Figure 2 shows 5 curves labelled A though E.

A is a full frequency sonic log plotted as interval velocity vs. two way time.

B is the corresponding full frequency reflection coefficients.

C is a low pass filtered version of B.

D is the velocity transform of C.

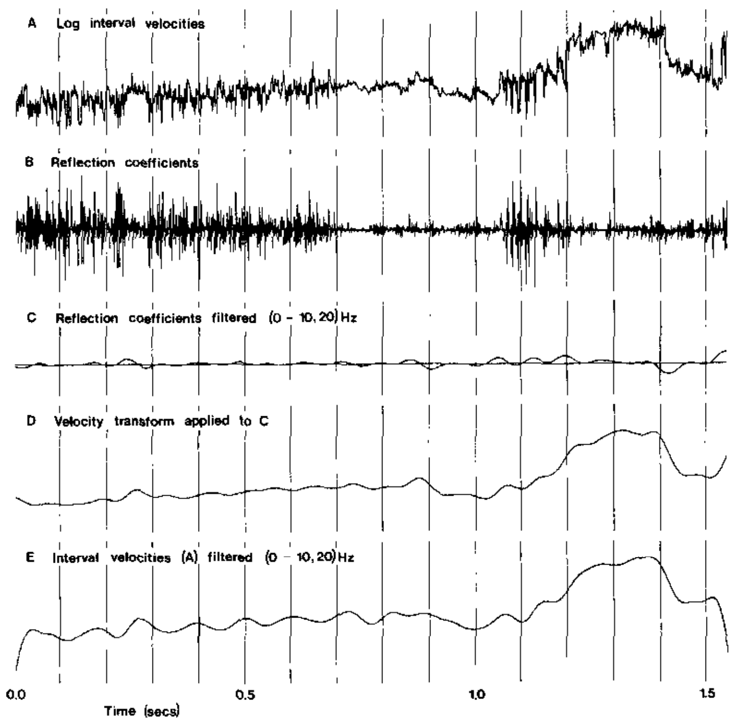


Fig. 2. Effect of mixing the use of filters and velocity transforms.

E is the low pass filtered version of A.

D & E are clearly not the same even though we started with the same data and applied the same filter. In the one case the filter is applied to the reflection coefficients and in the other case the filter is applied to the velocities. Then we made a comparison between the two by passing the former through a velocity transform. Clearly the velocity transform has distorted the filter response.

The point of all of this is that it is impotant to take care to be consistent about the domain in which filters are applied when dealing with the velocity transform.

THE SEISMOGRAM—HIGH PASS MODEL

The basic model of one trace of a seismogram is a set of reflection coefficients convolved with a wavelet. Of course, a real seismogram will be contaminated by random noise, multiples, diffractions and reflections out of the plane of the section. However, if the section is migrated, multiple attenuated and correctly deconvolved then the seismogram trace is a reasonable approximation to the reflection coefficients in the frequency band of the input data. The input seismic signal is always bandlimited and rarely contains any useful information in the region below 10 Hz. or above 100 Hz. because of the *Earth response, geophone and instrument* response, etc., and so even after correct deconvolution the seismogram trace only represents a bandpass version of the reflection coefficients.

A typical seismogram is shown in Figure 3. In Figure 4 we show its velocity transform. Because there is no low-frequency component present in the seismogram the velocity appears to oscillate about a constant mean value. This mean value is the z_0 of equation (6) and is chosen in this case arbitrarily. This appearance is quite different from the typical block-like appearance of an acoustic log. See, for instance, Figure 5. We would like the output of our velocity transform to have the appearance of an acoustic log and although we can never expect

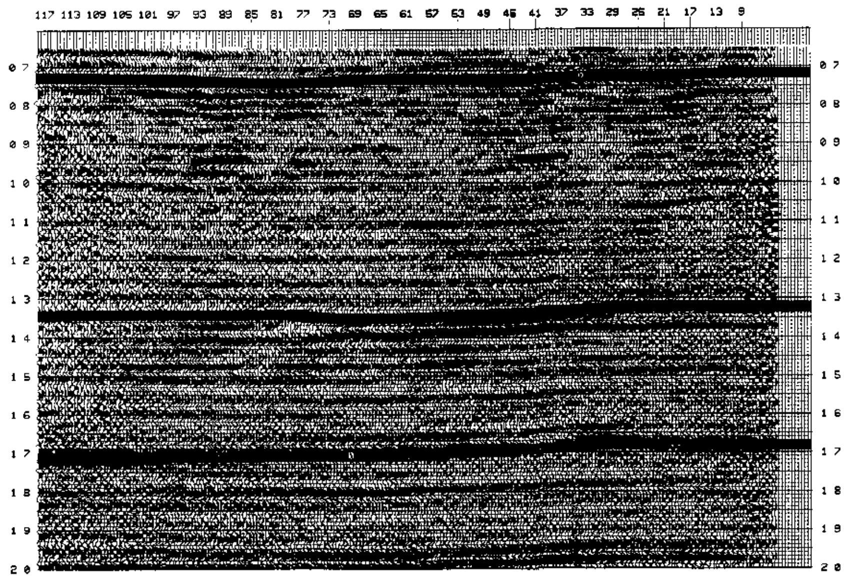


Fig. 3. Seismic section with 2 events picked, at about 1.3 and 1.7 seconds.

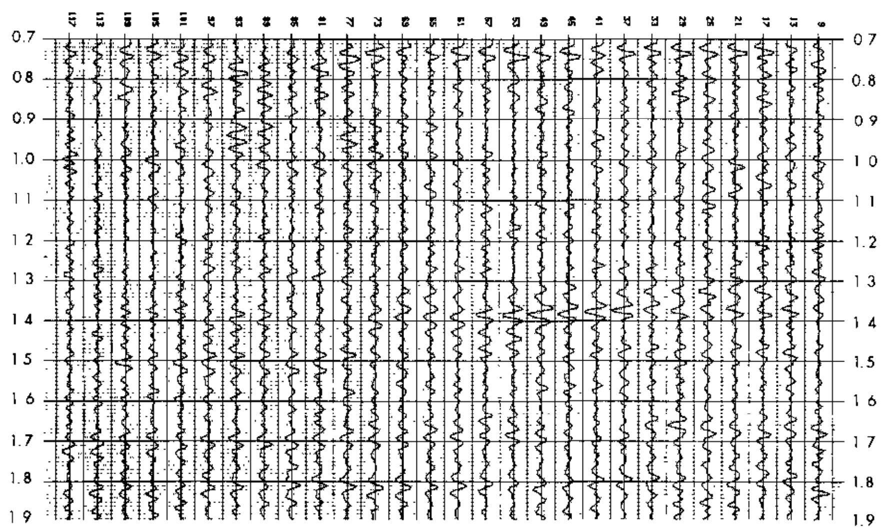


Fig. 4. Pseudo acoustic impedance logs derived from seismic section in Figure 3, using no low frequency information.

to get exactly to a sonic log, because the seismic bandwidth is only a few tens of Hz. whereas the log bandwidth is typically kilo Hz., we should be able to do better by trying to fill up the low frequency end of the spectrum. Then our velocity transform should represent a low-pass version of an acoustic log rather than a band-pass version.

Candidates For Low Frequency Velocities

There are three possible ways to recover the low frequency velocity information.

(i) We could take the velocities that were used to stack the seismic data.

(ii) We could attempt to build a geological model based on the seismic information and then select velocities for the formations in the model.

(iii) We could use acoustic logs of wells drilled on or near the seismic line.

Figure 5 shows these three candidates plotted as interval velocity versus two way time. The low frequency version of each is derived by converting to reflection coefficients, running a low-pass filter followed by a velocity transform.

The stacking velocities, while entirely adequate for stacking purposes fail to show the detail of the sonic log, even though we are only considering low frequency versions of these functions.

The geological model requires a good deal of interpretive skill to generate, because essentially every real event of the seismic section must be assigned to a formation interface and velocities for those formations chosen. However, it can be used effectively in areas where there is no well control.

From the above discussion and a study of Figure 5 we conclude that the best candidate for the purpose is the sonic log.

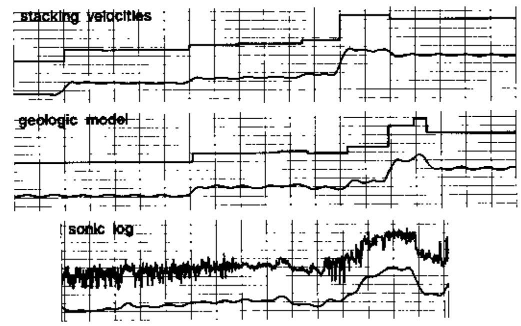


Fig. 5. Three possible candidates for supplying low frequency velocity information.

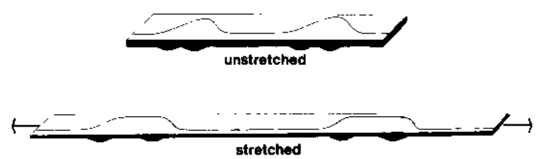


Fig. 6. Stretching a rubber sheet of uneven thickness — and the effect on a curve drawn on the surface.

A Stretching Algorithm

We will now discuss a method of stretching a sonic log so that it, in effect, will be the sonic log of a similar lithological sequence but having layers of different thicknesses. What we require is a way to stretch or compress the flat parts of the log and leave the steep, rapidly changing, parts alone.

An analogy to this process might go like this. We have a sonic log painted on a thin sheet of rubber. However, the rubber is not very uniform in thickness and in fact, wherever the log changes rapidly, that is, a steep gradient, the rubber is extremely thick and wherever the log is fairly flat in character — constant velocity — the rubber is thin.

So now we'll take this sheet and stretch it out — as shown in Figure 6.

The nice thing about this way of doing things is that what we've really done is to thin or thicken areas of constant velocity and leave the sudden changes in velocity untouched — they won't stretch so much because of the thick rubber.

Mathematically speaking we accomplish this effect by adding a stiffness factor to an ordinary linear mapping.

In Figure 7 we can see an example of the stretching algorithm and its obvious advantages over a linear method.

In practice, the algorithm is used to construct a sonic log at every trace position of the seismogram. To do this we proceed as follows:

(i) We select a number — about 6 or 7 usually — of clearly identifiable horizons on the seismogram and define them in terms of coordinates (trace number, two-way time). We call these coordinates the mapping picks.

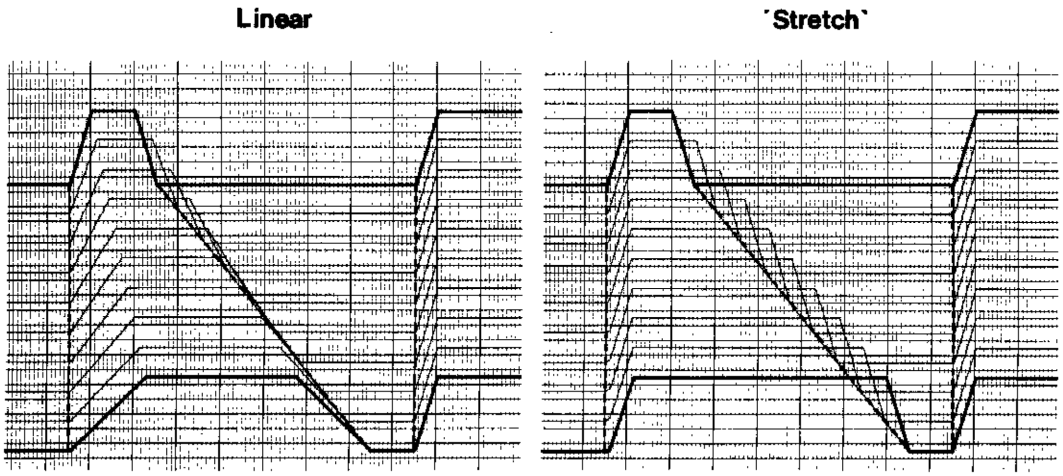


Fig. 7. Examples of a linear mapping algorithm versus the special 'stretch' algorithm.

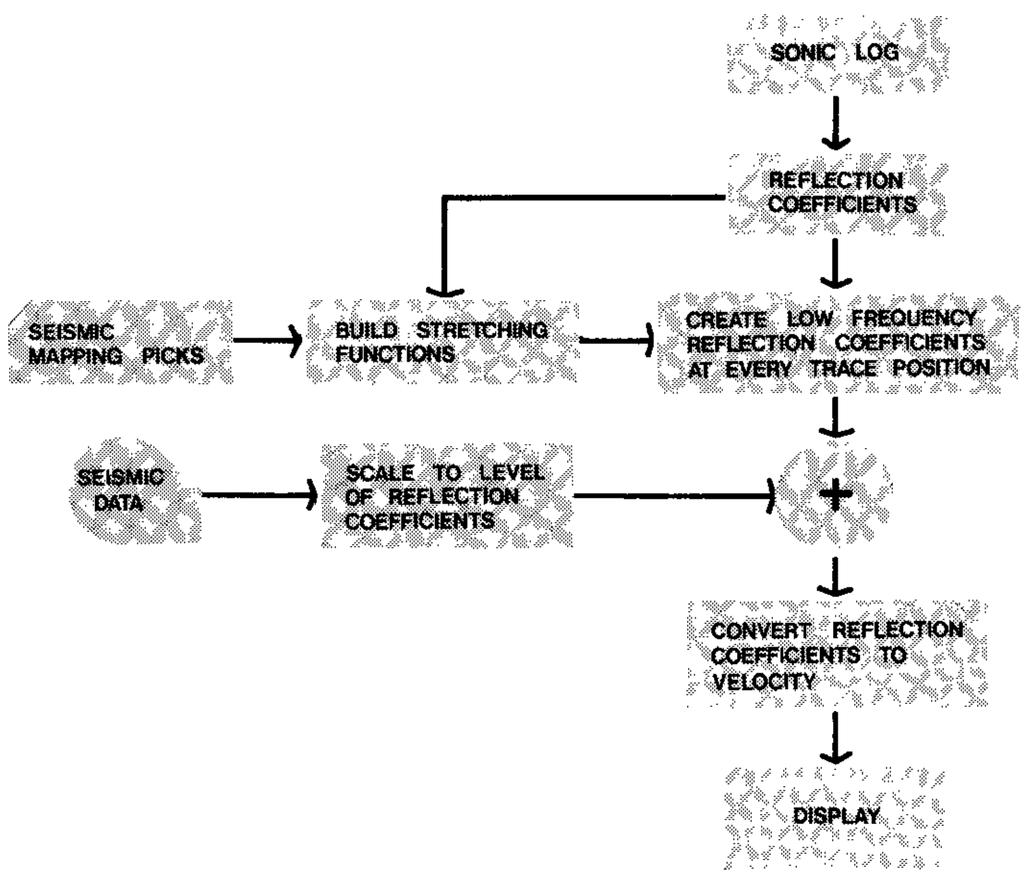


Fig. 8. Flowchart depicting a method of seismic inversion.

(ii) Next we form a list of the two-way times of these same events on the sonic log.

(iii) Now we stretch or compress the log according to the mapping picks.

(iv) Occassionally one or more events may pinch-out. This situation is handled by defining the location, etc. of the pinch-out from the seismogram. The appropriate portion of the sonic log is then removed before stretching is performed.

Note that we will only retain the low frequency portion of these 'stretched' logs and hence accuracy is not of paramount importance.

What we have now is a complete set of low-frequency seismic traces which we will combine with the ordinary seismogram to obtain our final result of acoustic impedance vs. two-way time.

Construction of the Acoustic Impedance Log

We will discuss the flow chart in Figure 8.

Starting at the top we take a sonic log and on the left a set of mapping picks and feed them into the box that will build a set of stretching functions — one function for each seismic trace positon. Next we apply these functions to the log, convert to reflection coefficients, and low-pass filter to get a complete set of low frequency reflection coefficients. Now we bring in our seismic data, scale it to the level of reflection coefficients and simply add each scaled seismic trace to the correponding low frequency reflection coefficient trace. The result can now be velocity transformed and displayed.

Some further points are worth mentioning. Strictly speaking we should start with acoustic impedances and not a sonic log, which is velocity only. So, if a density log is available, it should be multiplied by the sonic log. This will generate an acoustic impedance log which will then replace the sonic log in the flow chart.

The result of the process must always be considered as acoustic impedance.

The scale factor for the seismic data is definitely a problem which we chose to solve by comparing the energy in a tie trace to the energy of the sonic log in a window of both time and frequency.

The mapping picks, as previously mentioned, are arrived at by correlating horizons on the

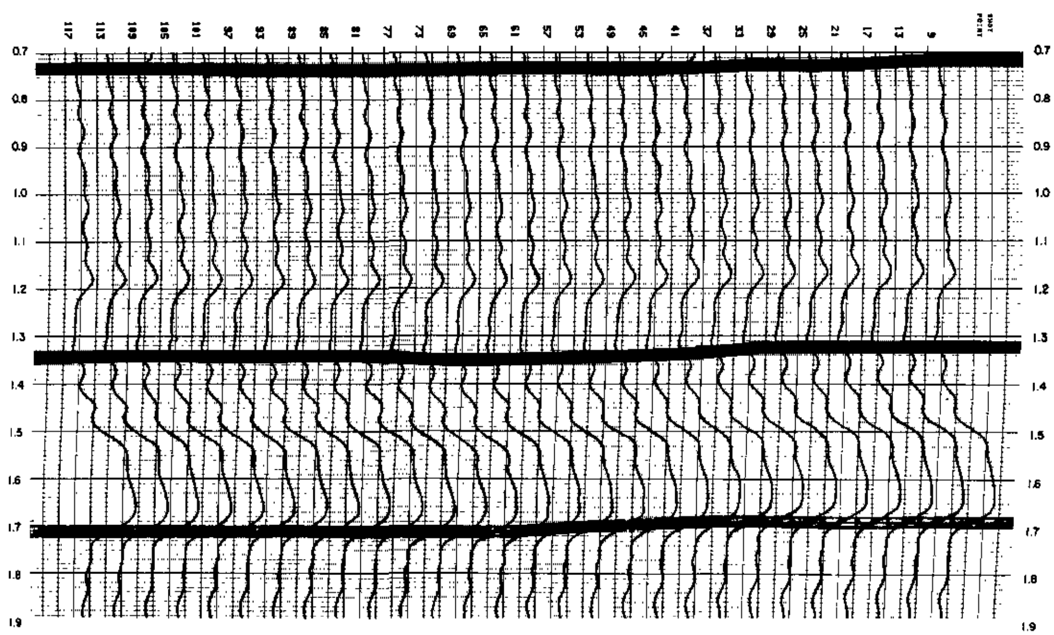


Fig. 9. Pseudo acoustic impedance logs derived from low-frequency velocity information only.

seismic section which can be easily identified on the sonic log. The stretching algorithm will then deal reasonably with intermediate events.

In Figure 9 we have shown the low frequency veocities. This figure was obtained by doing the stretching, etc. of the sonic log according to the mapping picks, converting to reflection coefficients, low pass filtering and finally performing a velocity transform.

Notice that in the flow chart Figure 8 we have a plus sign indicating addition of the low and high frequency reflection coefficients. Of course, if we used the velocities as shown in Figure 9, we would have to use multiplication as we pointed out earlier on — equation (8).

In Figure 10 we show the final result. The correlations at about 1.3 seconds and 1.7 seconds are simply to mark the same events as are shown on the seismogram in Figure 3.

Conclusion

We defined the velocity transform and looked at some of its properties. We saw that filtering must be performed with caution when velocities and reflection coefficients are being compared. We looked at seismic data and saw that there was a spectral hole to fill before we could properly derive velocities from seismic data. A stretching algorithm was defined and used on a sonic log to create low-frequency traces which filled the spectral hole. The completed velocity function was displayed versus two-way time.

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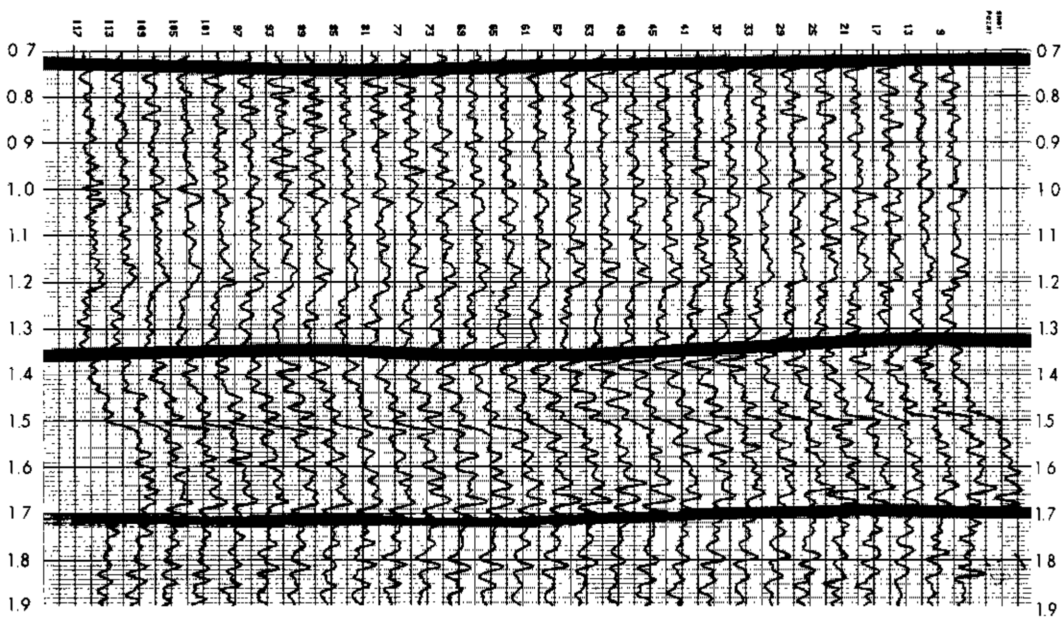

Fig. 10. Pseudo acoustic impedance logs derived by combining information in Figure 4 with that of Figure 9. Two events are picked at about 1.3 and 1.7 seconds for comparison with Figure 3.

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