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A FOURIER TRANSFORM MINIMIZATION TECHNIQUE FOR INTERPRETING MAGNETIC ANOMALIES OF SOME TWO-DIMENSIONAL BODIES[1]

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ABSTRACT

The paper deals with the Fourier transform analysis of magnetic anomalies to obtain depth(s) of burial of some two-dimensional magnetized bodies: a vertical fault, a two-dimensional dipping dike, a vertical sheet with finite or infinite depth extent and a horizontal circular cylinder. It is considered that the body is uniformly magnetized in the Earth's field. Simple formulas are derived which enable determination of depths to the different models. Prior knowledge of any geophysical parameters is not required in interpreting these anomalies.

The algorithm is developed in a compact indexed form which can readily be programmed on a computer. It is applied to synthetic data with and without random error for various test examples. It is also applied to two field problems, namely, over the Pima copper mine in Arizona (USA) and in the Kishanganj area in Bihar (India). The depths obtained are compared with drilling and seismic information.

INTRODUCTION

Estimation of depth of burial, dip, width, density and strength of magnetization from observed gravity and magnetic data are some of the prime objectives of interpretation in exploration geophysics. In oil exploration, this interpretation usually amounts to estimating the thickness of the sediments in basins. In mineral exploration, it is to find the depth of burial of ore bodies. Since the time of Peters's (1949) classic paper, many different approaches have been developed, and there have been numerous attempts to utilize computers for interpretation of gravity and magnetic data (Koulomzine et al., 1970; Hartman et al., 1971; Naudy, 1971; O'Brien, 1972; Hammer, 1977; Thompson, 1982; and Gupta, 1983a, b).

In the last few years, there has been a growing interest on the part of geoscientists to interpret potential-field data in the frequency domain where the inherent characteristics of the data are not altered. Fourier transform analysis, therefore, has emerged as an interesting and powerful tool for geophysical data interpretation. This has been used by Botezatu (1970) for the separation of gravity and magnetic anomalies, Kanasewich and Agarwal (1970) in analytic continuation, Agarwal and Lal (1971) for calculating the second derivative and Davis (1971) in the filtering process of field data. Odegard and Berg (1965), Gudmundsson (1966), Sharma and Geldart (1968), Bhattacharya (1971), Rao and Avasthi (1973) and Sengupta (1983) have used spectral analysis for estimating the depth of burial of various bodies.

The present paper deals with the Fourier transform analysis of some two-dimensional (2-D) magnetic anomalies, namely, those due to a vertical fault extending to infinity in the strike or y direction, a 2-D dipping dike with its normal section parallel to the xz plane, a vertical sheet with finite or infinite depth extent extending infinitely in the strike direction and a 2-D horizontal cylinder with its normal section parallel to the xz plane (Figure 1). It is considered that the body is uniformly magnetized in the Earth's field. Simple formulas are derived which allow determination of the depth(s) by minimizing certain mathematical expressions involving the field data in the frequency domain.

Present mathematical studies reveal that the Fourier transform minimization analysis of magnetic anomalies is sufficient to provide necessary geophysical information. It is also observed that prior knowledge of any geophysical parameters, such as susceptibility contrast, dip of the body, etc., is not required in interpreting these anomalies. The algorithm is developed in a compact indexed form suitable for efficient programming. It is applied on various test examples. Finally, it is applied to two field problems.

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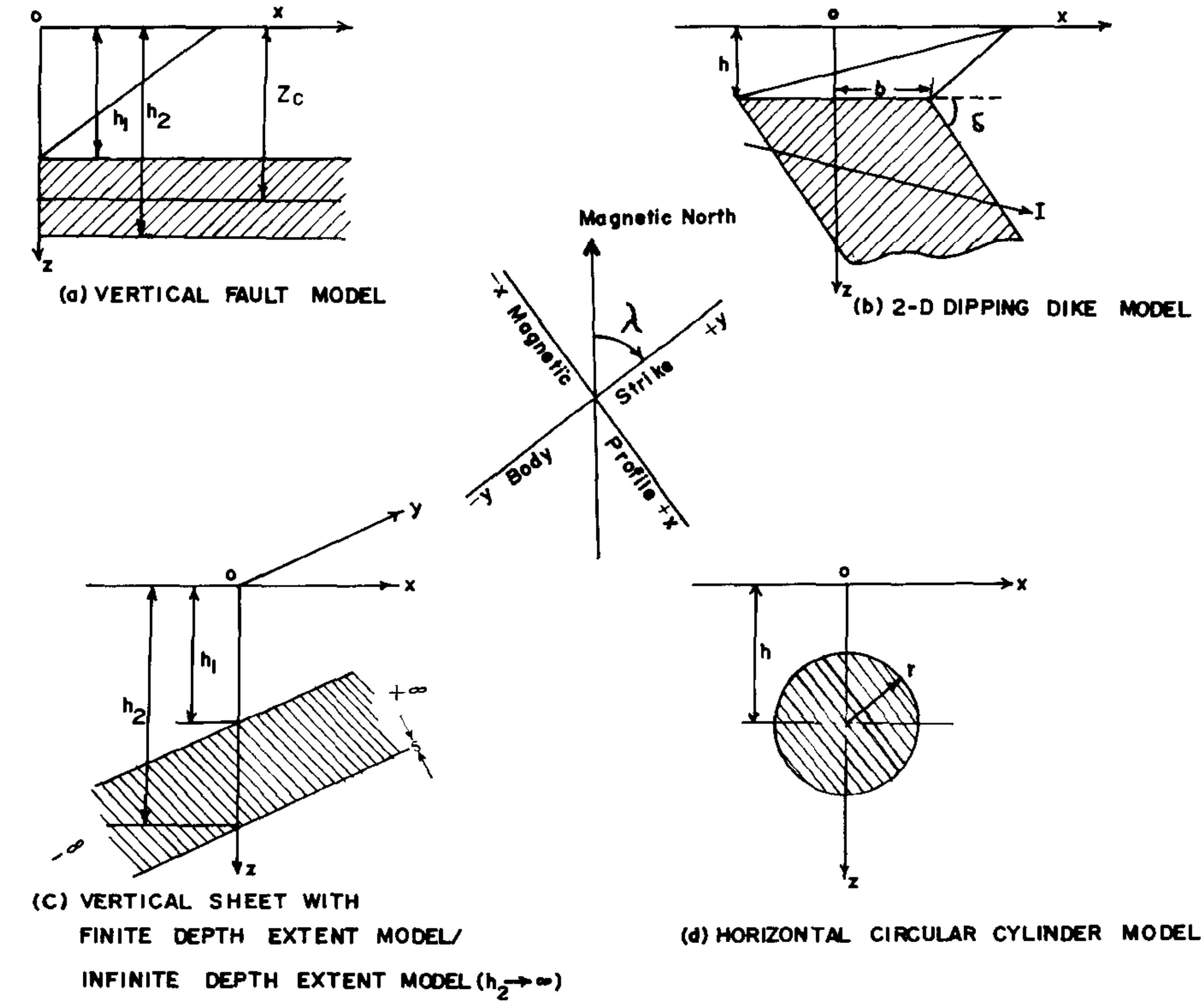

Fig. 1. *Diagrams for various simple geometrical structures.*

THEORY

Interpretation of isolated anomalies is usually based upon consideration of geometrically simple bodies. Such simple forms seldom match with the actual forms that geologic sources have. Nevertheless, the depths obtained under such assumptions are usually found to be in good agreement with the actual depths.

The magnetic effect, M (or anomalous magnetic field strength), due to a subsurface source observed at ground level, in general can be written as:

$$M_{mn}(x) = A_{mn}R_{mn}(x)\,, \tag{1}$$

where m = t, z, h for $M = \Delta T, \Delta Z, \Delta H$, respectively; n = 1, 2, 3, 4 for the four different anomaly-causing bodies: n = 1 denotes the vertical fault; n = 2, the dipping dike; n = 3, the vertical sheet with finite or infinite depth extent; and n = 4, the horizontal circular cylinder. A_{mn} are the amplitude coefficients and θ_{mn} [equations (2) to (5)] are the index parameters. These parameters are defined by Gay (1963, 1965), Grant and West (1965) and Atchuta Rao et al. (1980, 1983) for total field (ΔT), vertical field (ΔZ) and horizontal field (ΔH) and are given in Table 1.

$$R_{m1}(x) = (1/2)\ln\left[\left(x^2+h_2^2\right)/\left(x^2+h_1^2\right)\right]\cos\theta_{m1} + \left[\tan^{-1}\left(x/h_1\right) - \tan^{-1}\left(x/h_2\right)\right]\sin\theta_{m1}\,, \tag{2}$$

$$R_{m2}(x) = (1/2)\ln\left\{\left[(x+b)^2+h^2\right]/\left[(x-b)^2+h^2\right]\right\}\sin\theta_{m2} + \left\{\tan^{-1}[(x+b)/h] - \tan^{-1}[(x-b)/h]\right\}\cos\theta_{m2}\,, \tag{3}$$

n	m	Amplitude coefficient A_{mn}	Index parameter θ_{mn}
	t	$2kT_0\beta$	$2J-90°$
1	z	$2kT_0\sqrt{\beta}$	J
	h	$2kT_0\sqrt{\beta}\sin\lambda$	$J-90°$
	t	$2kT_0\beta\sin\delta$	$2J-\delta-90°$
2	z	$2kT_0\sqrt{\beta}\sin\delta$	$J-\delta$
	h	$2kT_0\sqrt{\beta}\sin\delta\ \sin\ \lambda$	$J-\delta-90°$
	t	$2kT_0\beta$	$2J-180°$
3	z	$2kT_0\sqrt{\beta}$	$J-90°$
	h	$2kT_0\sqrt{\beta}\sin\lambda$	$J-180°$
	t	$2kT_0\beta$	$2J-180°$
4	z	$2kT_0\sqrt{\beta}$	$J-90°$
	h	$2kT_0\sqrt{\beta}\sin\lambda$	$J-180°$

Table 1. Values of amplitude coefficient and the index parameter.

k = susceptibility contrast,
T_0 = normal level of the total magnetic field intensity,
I_0 = inclination of the normal geomagnetic field vector,
λ = strike of the body measured clockwise from magnetic north,
δ = dip of the body,
J = $\tan^{-1}(\tan I_0 / \sin\lambda) \equiv I_0'$ (inclination in the vertical plane containing Ox),
β = $1-\cos^2 I_0 \cos^2\lambda$.

$$R_{m3}(x) = s[(x\sin\theta_{m3} - h_2\cos\theta_{m3})/(x^2+h_2^2) - (x\sin\theta_{m3} - h_1\cos\theta_{m3})/(x^2+h_1^2)], \quad (4)$$

$$R_{m4}(x) = \pi r^2\left[(h^2-x^2)\cos\theta_{m4} + 2xh\sin\theta_{m4}\right]/(x^2+h^2)^2 . \quad (5)$$

The real and imaginary components of the Fourier transform of $M_{mn}(x)$ are defined as:

$$\mathrm{Re}F_{mn}(\omega) - i\,\mathrm{Im}\,F_{mn}(\omega) = A_{mn}\int_{-\infty}^{\infty} R_{mn}(x)e^{-i\omega x}dx, \quad (6)$$

where ω is the continuous spatial frequency in radians per unit length. The real and imaginary components of the Fourier transform of $M_{mn}(x)$ are

$$\mathrm{Re}\,F_{mn}(\omega) = A^*_{mn}\cos\theta_{mn}P_n(\omega) \quad (7)$$

and

$$\mathrm{Im}\,F_{mn}(\omega) = A^*_{mn}\sin\theta_{mn}P_n(\omega), \quad (8)$$

where

$$A^*_{m1} = \pi A_{m1}, \quad A^*_{m2} = 2\pi A_{m2},$$

$$A^*_{m3} = \pi s A_{m3}, \quad A^*_{m4} = \pi^2 r^2 A_{m4},$$

$$P_1 = \left(e^{-\omega h_1} - e^{-\omega h_2}\right)/\omega, \quad P_2 = e^{-\omega h}(\sin b\omega)/\omega,$$

$$P_3 = e^{-\omega h_1} - e^{-\omega h_2}, \quad P_4 = \omega e^{-\omega h},$$

for all positive values of ω.

In a field environment, the continuous function $M_{mn}(x)$ for particular values of m and n is sampled at different values of x. In any case, suppose there are altogether a total of N samples of $M_{mn}(x)$. In this case the Fourier transform of $M_{mn}(x)$ can be obtained using discrete Fourier transform (DFT) theory i.e.:

$$\mathrm{Re}F_{mn}(\omega_p) = \sum_{j=0}^{N-1} M_{mn}(j\Delta x)\cos(\omega_p\, j\Delta x), \quad (9)$$

$$\mathrm{Im}F_{mn}(\omega_p) = \sum_{j=0}^{N-1} M_{mn}(j\Delta x)\sin(\omega_p\, j\Delta x), \quad (10)$$

where $p = 0, 1, 2, \ldots, N-1$, Δx is the station interval, and $\omega_p = 2\pi p/(N\Delta x)$ are the discrete known spatial frequencies at which Re $F_{mn}(\omega_p)$ and Im $F_{mn}(\omega_p)$ are known.

Often in practice A_{mn} and θ_{mn} have to be estimated from geologic or other geophysical studies prior to interpretation. It is, therefore, essential to remove these parameters. By taking advantage of relations (7) and (8) and making use of relations (9) and (10), A_{mn} and θ_{mn} are eliminated and we obtain the following relations at $\omega = \omega_p$:

$$\frac{\mathrm{Re}F_{mn}(\omega_p)}{\mathrm{Re}F_{mn}(\omega_0)} = \frac{P_n(\omega_p)}{P_n(\omega_0)} = \frac{\mathrm{Im}F_{mn}(\omega_p)}{\mathrm{Im}F_{mn}(\omega_0)}, \quad n = 1,2, \quad (11)$$

$$\frac{\mathrm{Re}F_{mn}(\omega_p)}{\mathrm{Re}F_{mn}(\omega_1)} = \frac{P_n(\omega_p)}{P_n(\omega_1)} = \frac{\mathrm{Im}F_{mn}(\omega_p)}{\mathrm{Im}F_{mn}(\omega_1)}, \quad n = 3,4. \quad (12)$$

Further, from the above relations we obtain the following equations:

$$\mathrm{Re}F_{mn}(\omega_p) = \phi_{mn}(\omega_p, h_1, h_2), \quad n = 1,3$$

$$\mathrm{Re}F_{m2}(\omega_p) = \phi_{m2}(\omega_p, h, b),$$

$$\mathrm{Re}F_{m4}(\omega_p) = \phi_{m4}(\omega_p, h), \quad (13)$$

where

$$\phi_{m1} = \mathrm{Re}F_{m1}(\omega_0)\left[\left(e^{-\omega_p h_1} - e^{\omega_p h_2}\right)/\{\omega_p(h_2 - h_1)\}\right],$$

$$\phi_{m2} = \mathrm{Re}F_{m2}(\omega_0)\left[e^{-\omega_p h}\sin(\omega_p b)/\omega_p b\right],$$

$$\phi_{m3} = \mathrm{Re}F_{m3}(\omega_1)\left[\left(e^{-\omega_p h_1} - e^{\omega_p h_2}\right)/\left(e^{-\omega_1 h_1} - e^{-\omega_1 h_2}\right)\right],$$

$$\phi_{m4} = \mathrm{Re}F_{m4}(\omega_1)\left[\omega_p e^{-\omega_p h}/\omega_1 e^{-\omega_1 h}\right]. \tag{14}$$

The unknowns in equations (13), h_1, h_2, h and b, can be obtained by minimization with respect to the one or two unknowns, depending upon the case. Minimization of the equations (13) is expressed by the following:

$$\psi_{m1}(h_1, h_2) \equiv \sum_{p=0}^{N-1}\left[\mathrm{Re}\,F_{m1}(\omega_p) - \phi_{m1}(\omega_p, h_1, h_2)\right]^2, \tag{a}$$

$$\psi_{m2}(h, b) \equiv \sum_{p=0}^{N-1}\left[\mathrm{Re}\,F_{m2}(\omega_p) - \phi_{m2}(\omega_p, h, b)\right]^2, \tag{b}$$

$$\psi_{m3}(h_1, h_2) \equiv \sum_{p=1}^{N-1}\left[\mathrm{Re}\,F_{m3}(\omega_p) - \phi_{m3}(\omega_p, h_1, h_2)\right]^2, \tag{c) (15}$$

$$\psi_{m4}(h) \equiv \sum_{p=1}^{N-1}\left[\mathrm{Re}\,F_{m4}(\omega_p) - \phi_{m4}(\omega_p, h)\right]^2, \tag{d}$$

$$= \begin{cases} 0, & \text{for synthetic data without error,} \\ \text{minimum,} & \text{for data with error.} \end{cases} \tag{e}$$

These equations can be solved by the method of Fletcher and Powell (1963) or Fletcher and Reeves (1964).

In the case of a vertical sheet with infinite depth extent ($h_2 \to \infty$, $n = 3$), *equation (15c) can be further simplified* to give ψ_{m3} (h_1). Differentiating this simplified equation (15c) with respect to h_1, yields:

$$\sum_{p=0}^{N-1}\left[\mathrm{Re}\,F_{m3}(\omega_p) - \mathrm{Re}\,F_{m3}(\omega_0)e^{-\omega_p h_1}\right]\omega_p e^{-\omega_p h_1}$$

$$= 0. \tag{16}$$

Location of $x = 0$

In order to implement the theory developed in the previous section to field problems, the location of $x = 0$ (Figure 1) is required. For the case of the vertical fault, the 2-D dipping dike and the vertical sheet with infinite depth extent, the following mathematical formula can be used to obtain the value at $x = 0$:

$$M(0) = M_{\max} + M_{\min}, \tag{17}$$

where $M_{\max}$ and $M_{\min}$ are the maximum and minimum anomaly values along a profile. The algorithm developed by Banerjee (1972) can be used for other cases.

Test Examples and Discussion

The method of Fletcher and Powell (1963) was used to solve the equations (15). It is clear from Table 2 that the equations (15) give the exact values of h_1, h_2, h, and b when using data without error. After adding ± 2 percent random error into the synthetic data, the depth obtained is within ± 2 percent in the case of the horizontal cylinder and ± 8 percent / ± 6 percent in the case of vertical sheets with finite/infinite depth extent. In the case of the vertical fault, h_2 is approximately equal to h_1 $(1 + \varepsilon)$ where ε is of the order of 10^{-4}. Comparing the computed depths with the actual h_1 and h_2, the percentage error is large. However, if we compare this computed depth with $Z_c = (h_1 + h_2)/2$ (Figure 1), the percentage error is within ± 4 percent. This indicates that the throw of a fault can not be determined if the data contain error. In the case of gravity data analysis, the same conclusion was drawn by Gupta (1983b). In the case of a 2-D dipping dike, the depth can be obtained within ± 15 percent. However, the width b can not be determined when the data contain error. Nevertheless, b may be approximated from the following formula:

$$b \cong \sqrt{3}\, h \sqrt{\left[1.0 - \pi h M_{m2}(0) / \mathrm{Re}\, F_{m2}(\omega_0)\right]}. \quad (18)$$

This formula can be derived from the relations (1), (3) and (7). The computed b values are listed in Table 2.

Model No. n	Model depth (km)	Computed Depths in km: Using synthetic data without error	Using synthetic data with ±2% random error	Percentage error
		A_{mn} = 100 units	θ_{mn} = 30°	
(1) h_1	1.0	1.0	1.4418	44.18
z_c	1.5	1.5	1.4418	−3.97
h_2	2.0	2.0	1.4418	−27.90
(2) b	0.5	0.5	0.3878*	−22.44
h	2.0	2.0	2.2895	14.47
(3) h_1	1.0	1.0	1.0737	7.37
h_2	2.0	2.0	2.1176	5.88
(4) h	3.0	3.0	3.0534	1.78

Table 2. Theoretical results obtained by solving equations (15) for various models.

*In this case formula (18) was used.

Field Examples

Two field examples are considered. The first example (Figure 2) is taken from Gay (1963) in connection with a geological discovery in Arizona (USA), the Pima copper mine. The calculated depth is found to be 194 ft (59.1 m) while the actual depth as confirmed by drilling is 209 ft (63.7 m); Gay predicted the depth to be 229 ft (69.8 m). We would have, perhaps, acquired a more improved result if we could have dealt with the original data. The data used were obtained from the published figure (Gay, 1963) with the help of a digitizer.

The second example (Figure 3) is the vertical magnetic anomaly observed near Kishanganj in Bihar (India). The depth to the body is found to be 3.68 km while seismic study estimates the value to be of the order of 3 to 4 km in the area. Drilling data are not available.

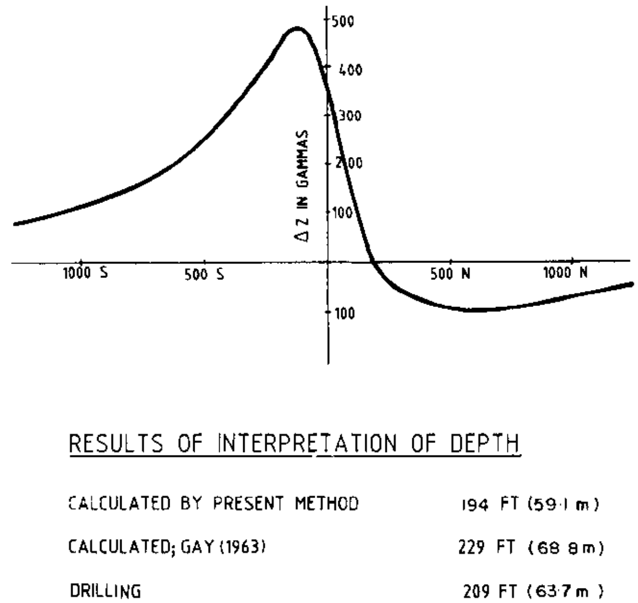

Fig. 2. Interpretation of a vertical magnetic anomaly over the Pima copper mine in Arizona.

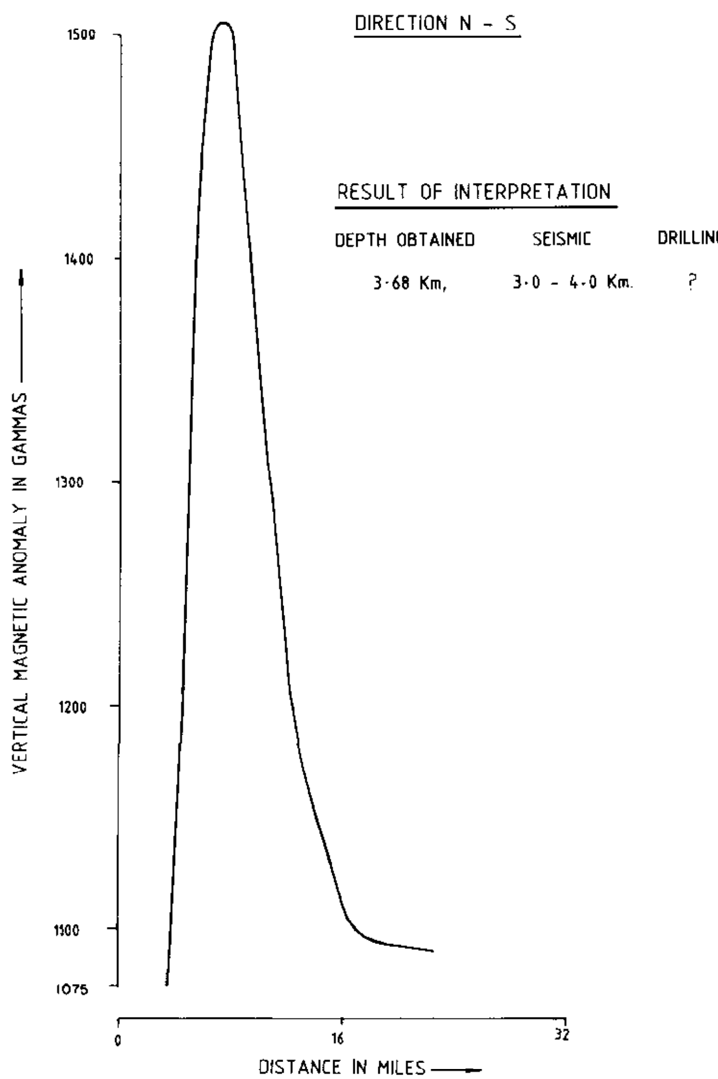

Fig. 3. Interpretation of a vertical magnetic anomaly over the Kishanganj area, India; 32.0 mi = 51.5 km.

Conclusions

It is shown that the Fourier transform minimization technique is a powerful tool for depth estimation. The potential advantage of this method is that it serves as an additional hand to an interpreter when the geophysical parameters are not known in advance. The algorithm is presented in a compact indexed form suitable for efficient programming. Similar analysis can be extended to the gradients of the total field, vertical field and horizontal field; and the vertical gradient of the Earth's magnetic field can be practically measured (see Barongo, 1985).

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