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SHORT NOTE

TOTAL MAGNETIC FIELD REDUCTION — THE POLE OR EQUATOR? A MODEL STUDY

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INTRODUCTION

A positive density contrast located below the observation plane causes a positive gravity anomaly centered above the highest point. On the other hand, a magnetized body causes a magnetic total-field anomaly which, in general, is positive over the equatorial edge of the body and negative over the polar edge. Thus, for each source body, there are two sets of contours on a magnetic map. The distance between the positive and negative extremes depends upon the depth, shape and dimensions of the source as well as on the inclination of the magnetic field. Near the poles, the magnetic anomaly due to a semi-infinite vertical prism is unipolar, i.e., there is only the positive component (on the north pole) centered over the body. On the equator, there is a negative component centered over the body with relatively small positive "lobes" on the northern and southern edges. At other latitudes, each body causes an anomaly with a positive and a negative component whose relative magnitudes and locations depend partially on magnetic field inclinations.

Baranov (1957) described a technique for reducing the maps made anywhere, except at very low latitudes, into what they would be if the inclination of the magnetic field were 90 degrees. These maps are called "reduced to the pole" maps and the process is called reduction to the pole (RTP). Over the years, several refinements and other methods of RTP, such as techniques using Fourier transforms (Bhattacharyya, 1966; Spector and Grant, 1970) have been suggested. Leu (1981) proposed reduction to the equator (RTE) and demonstrated that RTE is more reliable at high latitudes than RTP is at low latitudes.

In this note, I give equations for designing these operators and describe a model study to compare RTP (using conventional Baranov and Fourier transform operators) with RTE (using only Fourier transform operators). It is shown that RTE is more accurate than RTP even at high latitudes and that the Fourier transform provides more accurate RTP at lower latitudes than a conventional operator.

Although this study was not intended to determine the best of many methods of reduction to the pole or equator, it may be pointed out that Roy and Aina (1986) derived different equations using a slightly different derivation than Bhattacharyya (1966) and Gunn (1975). I found that Roy and Aina's operators were not as successful in transforming the synthetic fields as Bhattacharyya's operators used in this study. Silva (1986) treated RTP as an inverse problem and described a procedure for reducing the magnetic field to the pole at very low latitudes. Unfortunately, I could not duplicate the results of Silva's synthetic studies and, therefore, did not apply his methods on models discussed in this note.

EQUATIONS FOR THE REDUCTION PROCESS

1. RTP by Baranov's method

Starting from basic equations for Newtonian and magnetic potentials, Baranov (1957) derived the following equation for RTP in polar coordinates:

$$g' = -\mu T(O) - \frac{1}{2\pi}\int\int T(\rho,\omega)\Omega_3(\omega)\frac{d\rho}{\rho}d\omega \qquad (1)$$

where, following Baranov's notation:

$$\Omega_3(\omega) = 2\sum_{k=1}^{\infty}(-\eta)^k k(k+\mu)\cos k\omega$$

g'	=	total magnetic field at point O, reduced to the pole;
μ	=	$\sin I$, I being the inclination of the observed magnetic field;
$T(\rho, \omega)$	=	total field at the magnetic observation point (ρ, ω) (polar coordinates) with reference to a calculation point at the origin where the observed field is $T(O)$; and
η	=	$(1-\lvert\sin(I)\rvert)/\cos(I)$.

Equation (1) can easily be programmed to devise a two-dimensional operator to be applied to a grid.

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2. Fourier transform method

Bhattacharyya (1966) derived an equation for the Fourier transform of a semi-infinite rectangular prism for any magnetic field inclination or declination. Since any natural body can be considered a combination of appropriate semi-infinite rectangular prisms, it follows that the spectrum for any given set of sources can be simulated by a combination of such prisms. Thus, an operator designed to modify the magnetic field due to one such prism can be applied to any observed field to achieve a similar modification. Gunn (1975) gave the equations for transformation of magnetic fields in the frequency domain. From Gunn's equations, the following have been elaborated for the multiplicative filters or reduction operators O_P (for RTP) and O_E (for RTE):

$$O_P(u,v) = \frac{(u^2+v^2)\left\{n^2(u^2+v^2) - (lu+mv)^2\right\} - 2j(u^2+v^2)^{3/2}\, n(lu+mv)}{[(lu+mv)^2 + n^2(u^2+v^2)]^2}$$

and

$$O_E(u,v) = \frac{-(lu+mv)^2\,[\left\{n^2(u^2+v^2) - (lu+mv)^2\right\} + 2jn(u^2+v^2)^{1/2}(lu+mv)]}{[(lu+mv)^2 + n^2(u^2+v^2)]^2}$$

where l, m, n are direction cosines of the earth's field vector which is assumed to be parallel to the polarization vector (no remanent magnetization), u and v are Cartesian spatial angular-frequency coordinates, and $j = \sqrt{-1}$.

The reduction operators O_P and O_E can be used in two ways:

1. Compute the Fourier transform of the observed field, apply O_P (or O_E) on the transformed field and compute the inverse Fourier transform. This approach is more accurate but can only be applied to relatively small areas.
2. Compute O_P (or O_E) for various values of u and v for the inclination and declination of the magnetic field and compute its inverse transform. This gives the space-domain operator which can be convolved with the observed magnetic field. Inaccuracy creeps into this approach because the operator has to be truncated to manageable dimensions. However, the operator can be applied to maps of any size. In this model study, the second approach was followed. The grid size of the designed operators was 21 × 21 grid points. These operators are referred to as "Fourier operators" in the rest of this note.

Model Study

Figure 1a shows the magnetic field due to four rectangular semi-infinite prismatic bodies located as shown with their tops at depths of 0.625, 1.25, 2.5, and 7.5 grid interval units. The magnetic field has an inclination of 45 degrees and a declination of 120 degrees as shown. Figures 1b and 1c show the calculated magnetic fields for these bodies at the pole and the equator. Figures 2a and 2b show the field in Figure 1a after RTP using Baranov and Fourier operators respectively. Figure 2c shows the RTE field.

Figure 3a shows the magnetic field due to two prisms identified as A and B. Prism A has vertical sides while the other has sides sloping at 45 degrees away from the axis. Both prisms are located at depths of 5 grid units to the top and 15 grid units to the bottom. The inclination of the magnetic field is 15 degrees and declination 120 degrees. Figures 3b and 3c show the magnetic fields at the pole and at the equator. Figure 4 (a, b and c) shows RTP with Baranov's operator, RTP with the Fourier operator and RTE, respectively. Figure 5 (a, b and c) shows the same models as Figure 3 (a, b and c) except that the inclination is 75 degrees.

The following conclusions can be derived from a careful examination of these models:

1. RTP is adequate using either operator over those bodies that have vertical sides. For sloping sides, distortion is introduced along the (magnetic) northern edge of the body. Both operators can handle a large depth range. The Fourier operator seems to distort the anomalies less than the Baranov operator at lower latitudes (Figure 4) but at higher latitudes the Baranov operator is more accurate (Figure 5).
2. Negative anomalies are introduced around the bodies by both types of RTP operators at all inclinations. There does not seem to be any reason to pick either type of operator on this basis.
3. RTE, though not exact, is generally more accurate in terms of location and the shape of the anomalies for *all three models*.
4. RTE is as accurate for a sloping-side prism as for a vertical-side prism. This is not the case for RTP.
5. The same operator can handle a large range of depths for all operators.
6. The total magnetic field at the equator is not as simple as it is at the pole. However, the "rim anomalies" are much less pronounced on RTE maps than in total-field maps at middle latitudes. In addition, on RTE maps the bodies directly underly the anomalies. Due to the limitations of the process, RTP maps have serious and

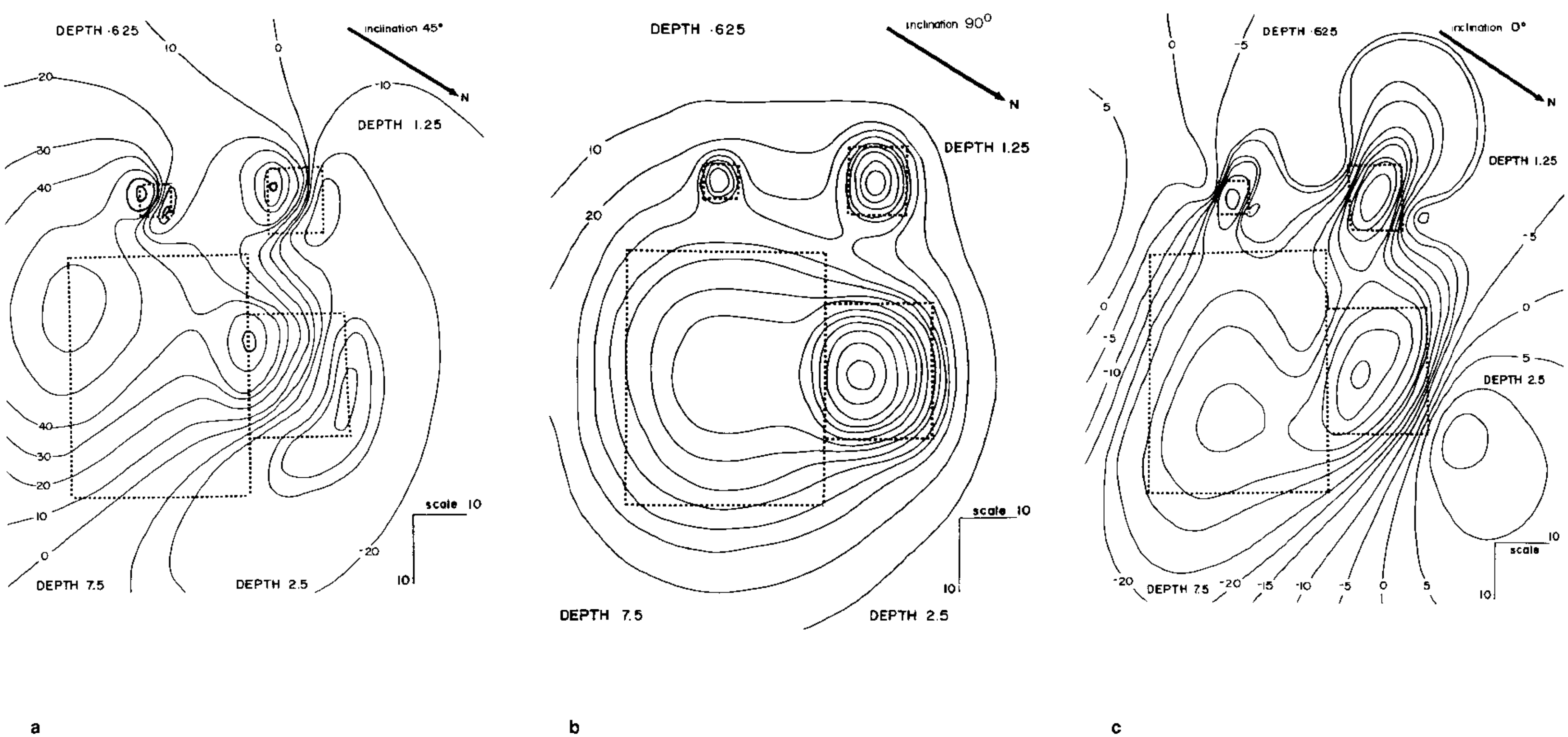

Fig. 1. Total field due to four semi-infinite square cylinders: **(a)** when magnetic field inclination is 45°; **(b)** at the pole; **(c)** at the equator. Relevant parameters are shown in the Figure.

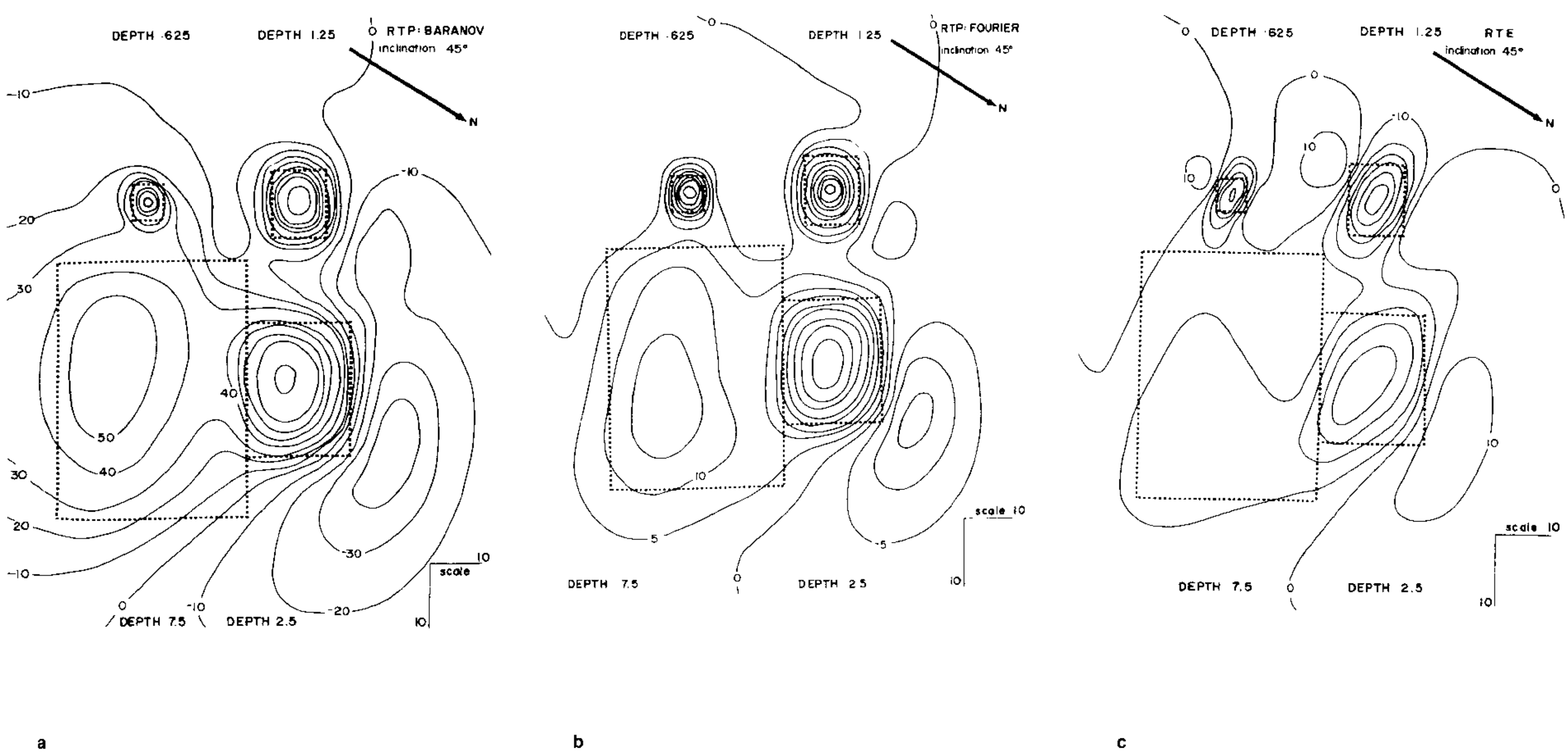

Fig. 2. Total field in Figure 1a after: **(a)** reduction to the pole using Baranov's operator; **(b)** reduction to the pole using the Fourier operator; **(c)** reduction to the equator.

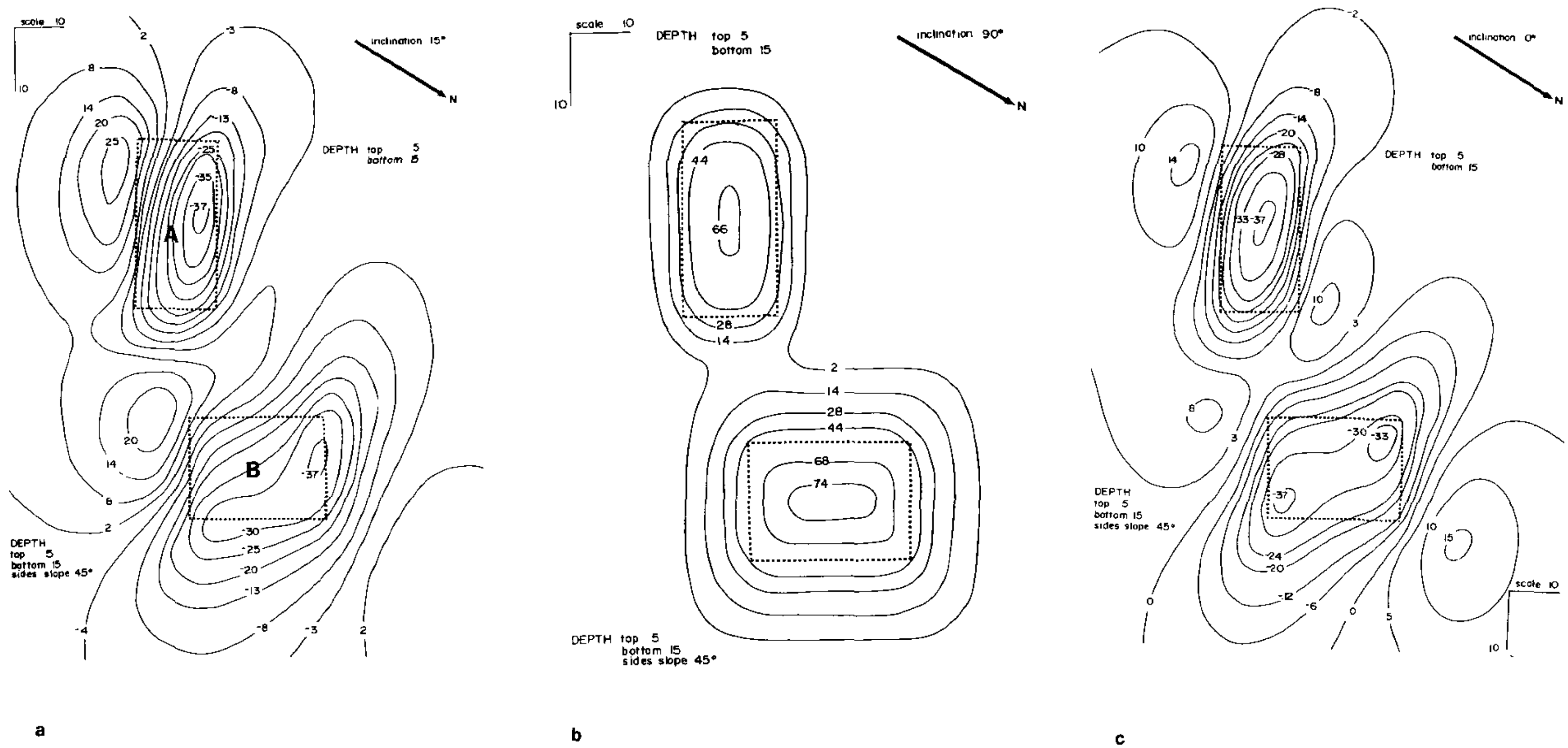

Fig. 3. Total field due to two prisms; **(a)** when the inclination of magnetic field is 15°; **(b)** at the north pole; **(c)** at the equator. Note that prism B has sloping sides.

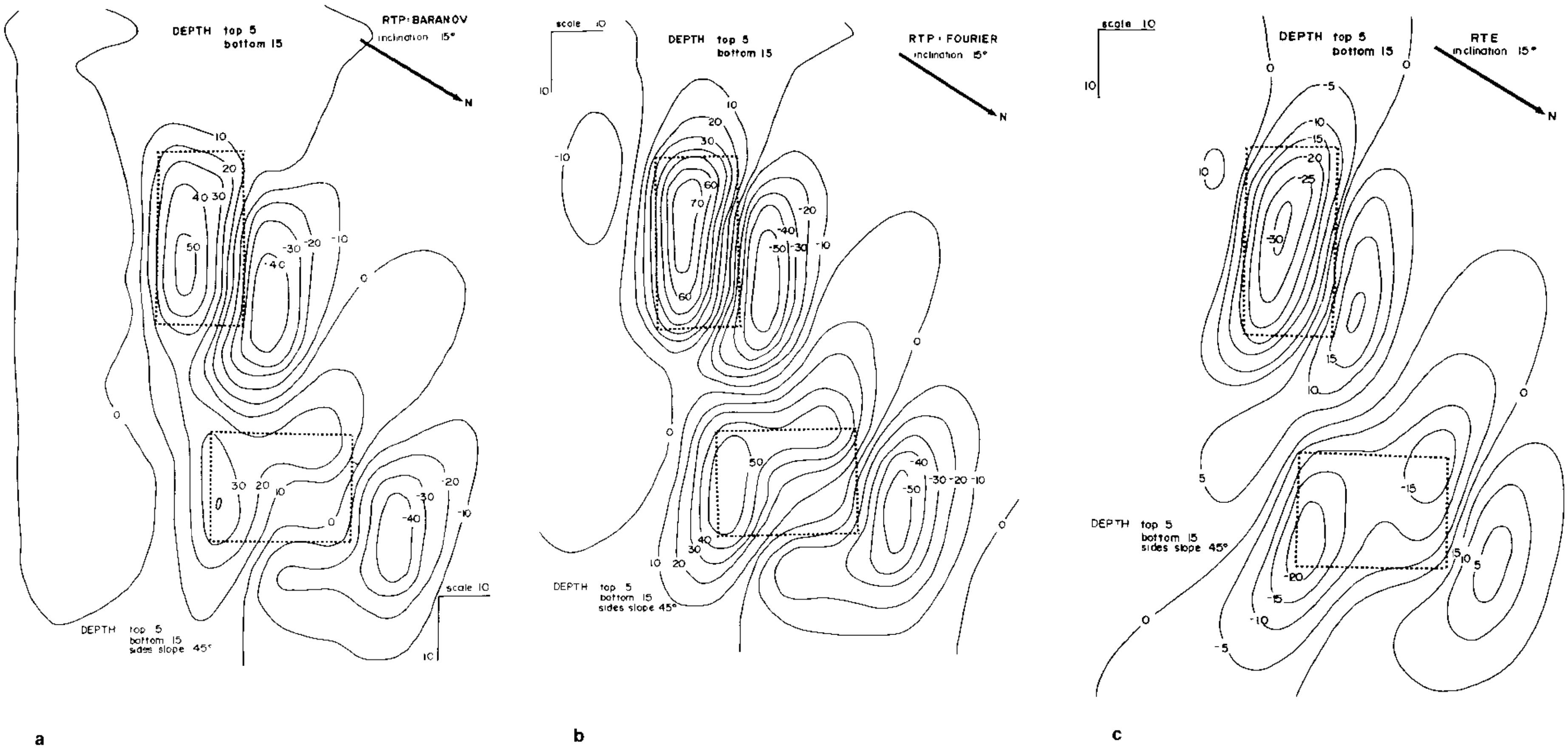

Fig. 4. Total field in Figure 3a after: **(a)** reduction to the pole using Baranov's operator; **(b)** reduction to the pole using the Fourier operator; **(c)** reduction to the equator.

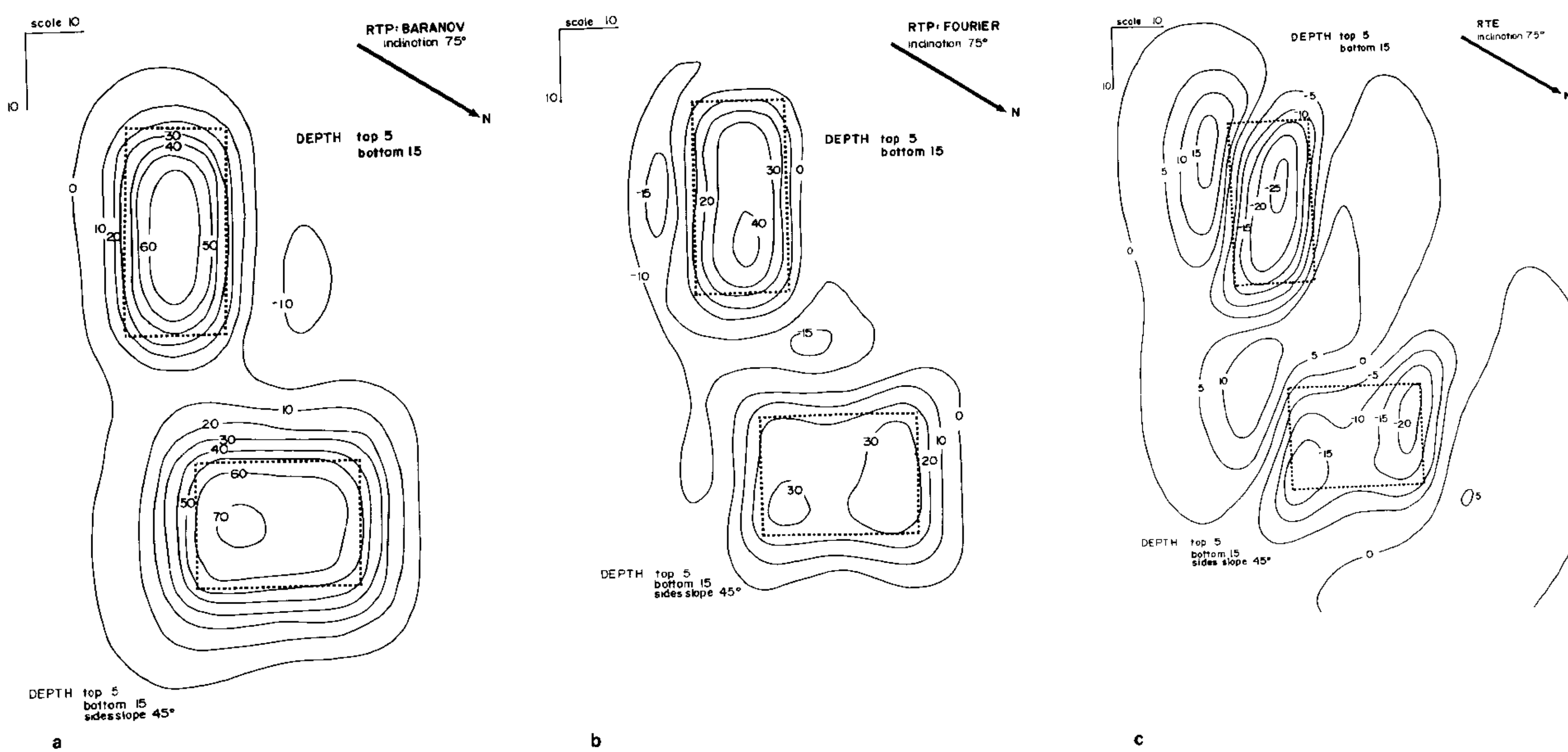

Fig. 5. Total field for two prisms when inclination is 75° after: **(a)** reduction to the north pole using Baranov's operator; **(b)** reduction to the north pole using the Fourier operator; **(c)** reduction to the equator.

misleading "rim anomalies" which are weaker in RTE maps and, in any event, are expected to be there. Therefore, the models show that RTE can be achieved with less harmful side effects than RTP.

SUMMARY

The model studies presented here show that even though the magnetic field is more complex at the equator than the actual magnetic field at the pole, a "reduced to the equator" map is less complex and more accurate than a "reduced to the pole" map.

In general, reduction to the equator is preferable to reduction to the pole, more particularly at the middle and lower latitudes. Except at very high latitudes, the Fourier operator is preferable to the conventional Baranov operator.

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