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PREDICTION OF SHALLOW SUB-BOTTOM SEDIMENT ACOUSTIC IMPEDANCE WHILE ESTIMATING ABSORPTION AND OTHER LOSSES

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ABSTRACT

A model is developed for estimating the acoustic absorption in marine sediments. The model correlates to Hamilton's (1972) *in situ* measurement in marine sediments. It is used to correct for losses in the bottom sediment so that the reflection coefficients and acoustic impedance of the sediments can be calculated as if they were surficial sediments. Results are given to show agreement with actual core data.

INTRODUCTION

During the last decade the ability to predict the geotechnical properties of surficial sediments from normal reflectivity has become established for engineering work (Breslau, 1964; Hamilton, 1970, 1971a; Hamilton *et al.*, 1970; Parrott *et al.*, 1979).

These authors have established relationships between the reflection coefficient and acoustic impedance and the geotechnical properties (porosity, density, grain size, bulk modulus, etc.) of the soil for various continental regions. In applying these findings, a reasonable number of surficial core samples are taken to adjust the numerical equations arrived at by each investigator.

In many cases, it is desirable to extend this technology to the identification of the sub-bottom layers. This paper delineates a technique for effectively removing the losses in the layers above the desired one, so that the desired layer can be analyzed as if it were a surficial reflector. This concept can be extended to each subsequent layer until the signal-to-noise ratio is at a level from which information cannot be extracted with accuracy (normally 5 db). In particular, a model is generated that compensates for the absorption and other losses in each layer as a function of frequency. This model is combined with the classical multilayer reflection mathematics developed by Officer (1958), to yield reflection coefficients equivalent to surficial sediments for the sub-bottom layers.

REVIEW OF SURFICIAL GEOLOGICAL PREDICTIONS

Hamilton, Breslau, Simpkin *et al.*, dating from the early 1960s, have established a clear trend for the relationship between reflection coefficients and soil interpretation; *i.e.*, clay, silts, sands, etc. In this discussion the acoustic reflection coefficient (R) is defined as:

$$R = \frac{(Z_s - Z_w)}{(Z_s + Z_w)} \quad (1a)$$

WHERE $Z_w = \rho_w C_w$ = WATER IMPEDANCE

$Z_s = \rho_s C_s$ = SOIL IMPEDANCE

$\rho_w = 1\ g/cm^3$

$C_w = 15000\ \frac{cm}{sec}$

It is clear that the acoustic impedance of the surficial layer can be calculated readily. Typical data generated by Hamilton (1969) are given in Table 1 relating impedances to sediment description.

DESCRIPTION	ACOUSTIC IMPEDANCE $\times 10^2\ \frac{g}{cm^2 s}$	SOIL LEGEND
WATER	1450	
SILTY CLAY	2016 - 2460	CLAY
CLAYEY SILT	2460 - 2864	SILT
SILTY SAND	2864 - 3052	SAND
VERY FINE SAND	3052 - 3219	
FINE SAND	3219 - 3281	
MEDIUM SAND	3281 - 3492	
COARSE SAND	3492 - 3647	
GRAVELLY SAND	3647 - 3880	
SANDY GRAVEL	3880 - 3927	

(AFTER HAMILTON (1969))
(CORRECTED FOR TEMPERATURE AND SALINITY)

Table 1. Soil classification versus acoustic impedance range.

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Therefore, normal reflectivity data combined with proper beam-width instrumentation and the historical data can be used to estimate the surficial acoustic impedance of the bottom sediments. A procedure is required to make the sub-bottom sediments appear as surficial, so that the above technique can be applied.

Absorption Model

The absorption loss in the sediments is the most critical factor in determining correct reflection coefficients; hence the acoustic impedance relative to sea water. Hamilton (1972) and Caulfield and Fass (1970) give experimental data for absorption in marine sediments. Figure 1 is a reproduction of Hamilton's (1972) data.

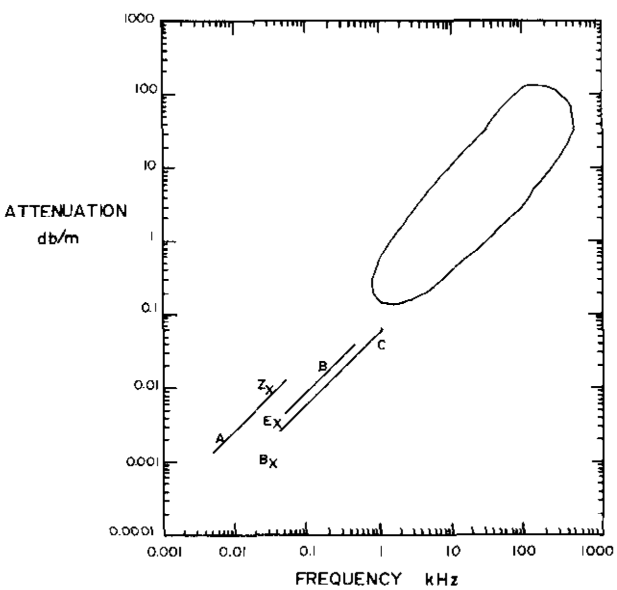

Fig. 1. Attenuation as a function of frequency (reproduction of Hamilton, 1972).

To generate a model for the absorption in marine sediments, it is assumed that the bottom consists of hypothetical layers of the order of one or two wavelengths in thickness that can be considered as potential barriers, as in the classical one-dimensional wave equation case. From Eisberg (1961), the equation for transmissibility with losses is:

$$T = T_0 e^{-AkX} \qquad (1b)$$

WHERE T = TRANSMISSIBILITY OF THE BARRIER $= \frac{E_T}{E_N}$

E_T = TOTAL ENERGY TRANSMITTED THROUGH THE BARRIER WITHOUT LOSS

E_N = ENERGY INCIDENT ON THE BARRIER

k = WAVE NUMBER IN THE BARRIER $= \frac{\omega}{C}$

ω = ANGULAR VELOCITY

C = VELOCITY

X = THICKNESS OF THE BARRIER

A = CONSTANT

It is assumed that the exponential term is a gross measure of the absorption in the layer.

Officer (1958) has shown that the absorption term has been found to be proportional to viscosity, frequency, sound velocity and density. In order to apply the loss equation to the acoustic model it must be dimensionally correct and reflect the above dependence. Therefore, the constant A in the loss equation has been assumed to have the following dependence:

$$A = \frac{\rho}{B} \qquad (2a)$$

WHERE ρ = DENSITY

B = DIMENSIONALITY CONSTANT REPRESENTING VISCOSITY, EXPERIMENTALLY DERIVED

Then the energy transmitted through the barrier with absorption is reduced to:

$$T = T_0 e^{-\frac{\rho\omega}{BC} X} \qquad (2b)$$

Figure 2 shows the same data as Figure 1 with the overlay of the absorption term as a function of frequency for sand and clay, utilizing Hamilton's data for density and velocity for each sediment. The value of the constant B is 161.8 from the experimental data acquired during the Polar Gas Survey (1974) application of this model. Table 2 is the numerical solution of the data in Figure 2. The agreement is reasonable considering the simplicity of the model, and is sufficient for obtaining geotechnical data for construction applications. Obviously, the accuracy of the model can be improved by using regional core data in selecting the constant B.

DENSITY = 1.42 SOUND VELOCITY = 1530		DENSITY = 2.42 SOUND VELOCITY = 1836	
FREQ (Hz)	ENERGY (dB)	FREQ (Hz)	ENERGY (dB)
1000	-0.1561	1000	-0.2223
2000	-0.3122	2000	-0.4446
3000	-0.4683	3000	-0.6669
4000	-0.6245	4000	-0.8892
5000	-0.7806	5000	-1.1115
6000	-0.9367	6000	-1.3338
7000	-1.0928	7000	-1.5561
8000	-1.2489	8000	-1.7784
9000	-1.4050	9000	-2.0006
10000	-1.5612	10000	-2 2229

Table 2. Relation between absorption loss and frequency (numerical solution of Fig. 2).

Other Losses

Since most shallow seismic surveys use point sources, one must correct for spherical spreading of the sound wave. The correction is the classical transmission loss equation:

$$N_W = 20 \log(R) \qquad (3)$$

WHERE R = RANGE FROM SOURCE TO LAYER OF INTEREST

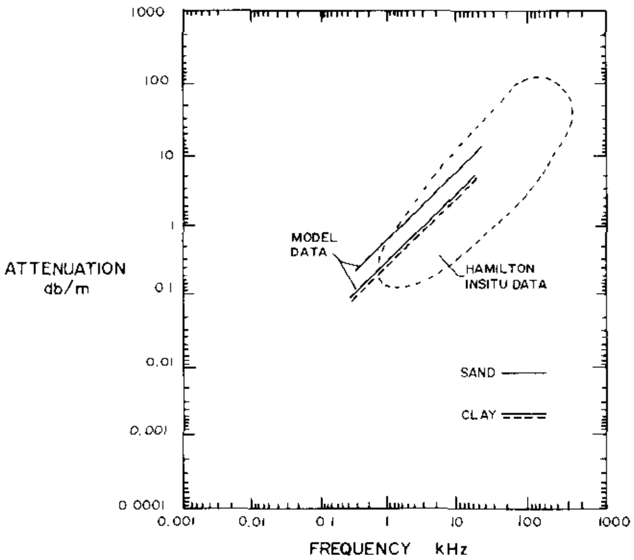

Fig. 2. Model attenuation prediction versus Hamilton (1972) *in situ measurements*.

Further, in the real-world application the model must correct for source and receiver directionality, or beam-pattern phenomena. This correction is normally done by using the shallow seismic system manufacturer's specification. The resulting seismic signal is then as close as possible to an approximation of a plane wave being incident on the bottom.

PROCEDURES FOR IMPEDANCE CALCULATION VERSUS DEPTH

Figure 3 illustrates typical wave forms from a shallow sub-bottom record. The water-depth travel time (t_w) is shown as well as the sampling window (dt) for calculating the acoustic impedance versus travel time. As in deep seismics, all calculations are done with respect to travel time. After sediment velocities are known the predictions are corrected to impedance versus depth.

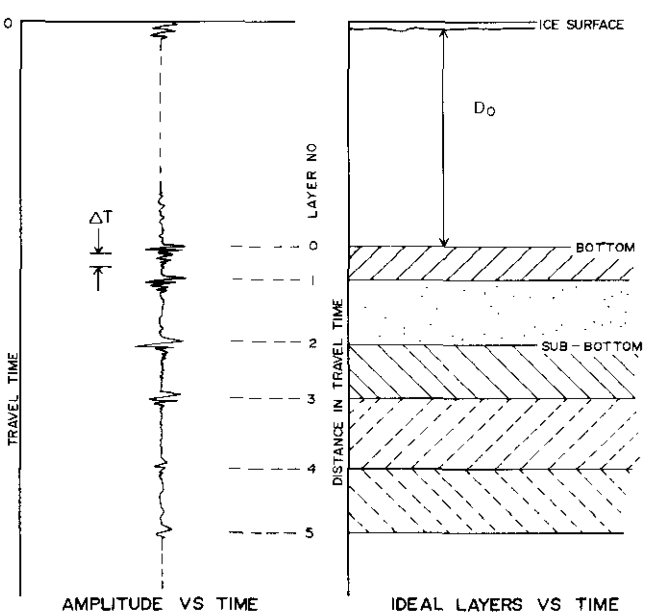

Fig. 3. Physical reflection model.

Referring all procedures to the travel time where the bottom echo starts [after duration of (t_w)], the sequential data are divided into N subsections, $S(dt_1)$, $S(dt_2)$, ... $S(dt)_N$. The (dt) period is selected to be 2 to 4 wavelengths to satisfy the absorption-model criteria. The sampling rate is sufficiently high that all sampling criteria such as Nyquist rate are satisfied.

The Fourir Transform to each subsection of the sequential data may be calculated as shown in equation (4):

$$F(\omega_m, t_n) = \sum_{t=t_1}^{t_2} S(t)\, e^{-i\omega_m t} \tag{4}$$

WHERE $t_1 = t_w + \sum_{i=1}^{n-1} \delta t_i$

$t_2 = t_1 + \delta t_n$

t_w = WATER DEPTH TRAVEL TIME

δt_n = SAMPLE WINDOW OF n^{th} SUB-SECTION

In this equation, $S(dt_n)$ signifies the n^{th} subsection of data window (dt_n), $S(dt_n) = S(t_1)$, ... $S(t_1 + dt_n)$.

Then the energy matrix

$$E(\omega_m, \delta t_n) = [F(\omega_m\ \delta t_n)]^2$$

$$E = \begin{bmatrix} E(\omega_1, \delta t_1) & E(\omega_1, \delta t_2) & \cdot & E(\omega_1, \delta t_n) \\ E(\omega_2, \delta t_1) & & & \\ \vdots & & & \\ E(\omega_m, \delta t_1) & & & E(\omega_1, \delta t_n) \end{bmatrix} \tag{5}$$

can be formed, yielding energy versus travel time where (W_m) is the n^{th} spectrum component. By utilizing the water-depth travel time as a reference, the matrix elements, $E(W_m, dt_n)$, are corrected for spherical spreading, as illustrated in equation (6). All frequencies are corrected with this factor.

$$N_{spercor} = 10 \quad 20\log(D_n) - 20\log(D_0) \tag{6}$$

WHERE $D_n = t_w \cdot C_w + \delta t_n \cdot C_{avg}$

C_w = WATER VELOCITY

C_{avg} = AVERAGE SEDIMENT VELOCITY (ASSUMED)

$D_0 = t_w \cdot C_w$ = WATER DEPTH

$E_{spercor}(\omega_m, \delta t_n) = E(\omega_m, \delta t_n) + N_{spercor}$

Finally, the absorption correction based on the model developed is applied. An initial assumption is made concerning the average density (ρ) and sediment velocity (C_{avg}), based on the duration of the bottom signal. A long-duration signal usually implies high clay content, a short signal a predominance of sand. The bottom-energy matrix corrected for absorption is found to be:

$$E_{cor}(\omega_m, dt_n) = E_{spercor} \cdot e^{\frac{\rho \omega_m}{BC} \cdot X_n} \qquad (7)$$

WHERE ρ = AVERAGE DENSITY

$\omega_m = 2\pi f_m$

$B = 161.8$

$X_n = \sum_{n=1}^{N} dt_n \; C_{avg}$

Now that the absorption correction for each frequency has been made, the total energy per layer ($E(dt_n)$) can be simply calculated by using equation (8).

$$E(dt_n) = \sum_{m=1}^{M} E(\omega_m, dt_n) \qquad (8)$$

WHERE M = TOTAL NUMBER OF FREQUENCY SAMPLES

It is now assumed that, since all losses have been accounted for, the sum of the energy in each layer equals the total energy incident on the bottom. Hence:

$$E_{total} = \sum_{n=1}^{N} E(dt_n) \qquad (9)$$

WHERE N = MAXIMUM NUMBER OF ENERGY SAMPLES. FOR THE ENERGY BEING GREATER THAN THE NOISE.

In practical applications, the statistical properties of the ambient and coherent noise must be removed from the data by conventional means before the preceding procedures are undertaken.

Now that the energy per layer and total energy incident are known, the reflection coefficient and resulting acoustic impedance of each layer with respect to water can be calculated.

The acoustic reflection coefficient of the first layer is the ratio of the reflected energy divided by the total energy incident on the bottom:

$$R_1 = \sqrt{\frac{E(dt_1)}{E_{total}}} \qquad (10)$$

The acoustic reflection coefficient of the n^{th} layer is the ratio of the energy from that layer divided by the total energy, minus the energy reflected from the previous layers:

$$R_n = \sqrt{\frac{E(dt_n)}{E_{total} - \sum_{m=1}^{n-1} E(dt_m)}} \qquad (11)$$

After reflection coefficients have been computed for the N layers, the sediment impedance for any layer, S, may be calculated:

$$Z_s = Z_w \frac{(1+R)}{(1-R)} \qquad (12)$$

WHERE Z_s = IMPEDANCE OF THE SEDIMENT RELATIVE TO WATER

Z_w = IMPEDANCE OF THE WATER

The acoustic impedance (Z_s) is plotted as a function of travel time for each (dt_n) increment in Figure 4.

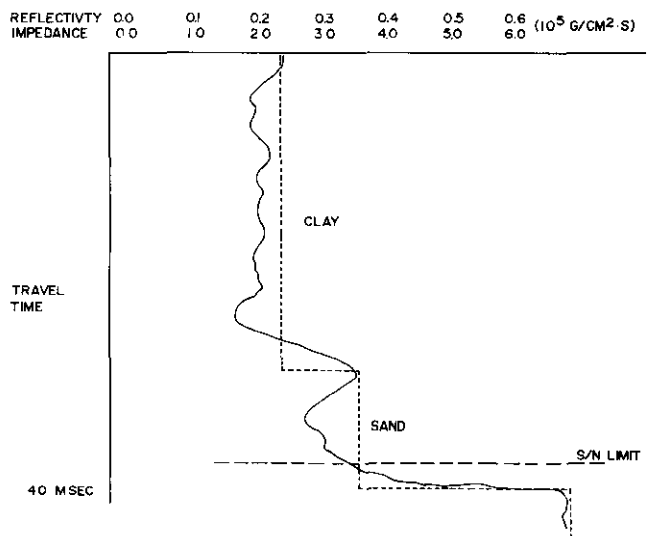

Fig. 4. Acoustic impedance as a function of travel time.

Layer classification is made according to Table 1, developed from Hamilton (1969) and Breslau (1964), and should include local core data to apply for corrections to the given geological area. Figure 4 also shows how the layers are classified by utilizing this table. The original model assumes that all layers increase in impedance as a function of depth. Research into handling layers of decreasing acoustic impedance by examination of the waveform phase is now under way. For most marine geotechnical applications, however, the original approach provides useful data.

Results

Many kilometres of shallow seismic data were acquired in the Beaufort Sea during the summer of 1981. A field computer was on the same vessel, recording and processing the data for acoustic impedance versus depth.

For this effort an average velocity of the sediment was used to convert the travel-time to depth. A typical boomer record with the field computer acoustic impedance overlay is shown in Figure 5.

During the course of the field efforts a number of cores were taken to verify the acoustic predictions. Figures 6 through 10 show the comparison between the computer predictions and the actual cores. The data were provided courtesy of Gulf Canada Resources. Figure 5 summarizes the relationship derived between acoustic impedance and soil classification for Beaufort Sea sediments.

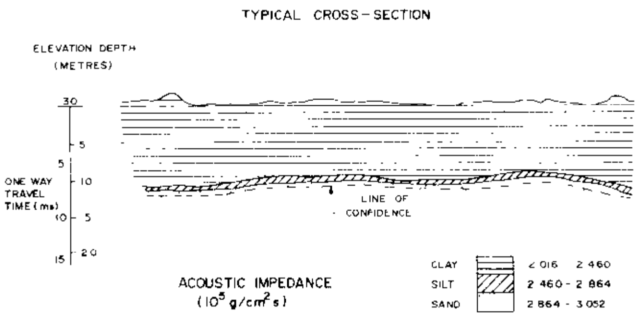

Fig. 5a. Acoustic impedance and soil classification for the sediment in the Beaufort sea.

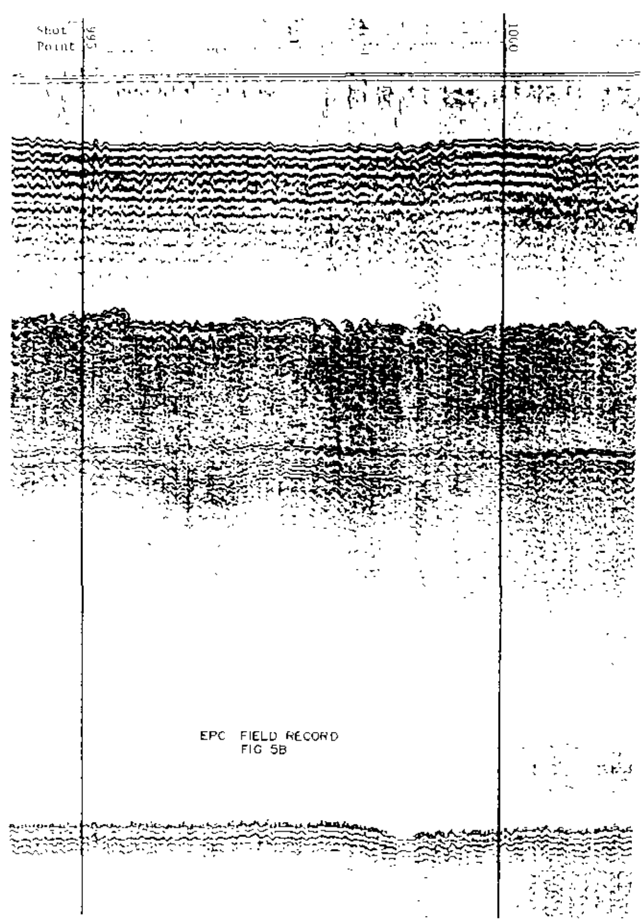

Fig. 5b. EPC field record.

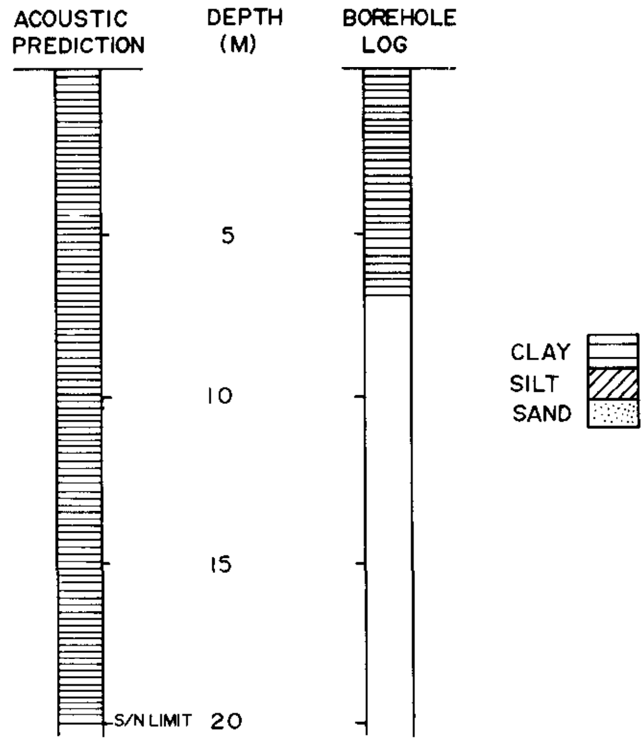

Fig. 6. Acoustic prediction versus borehole log.

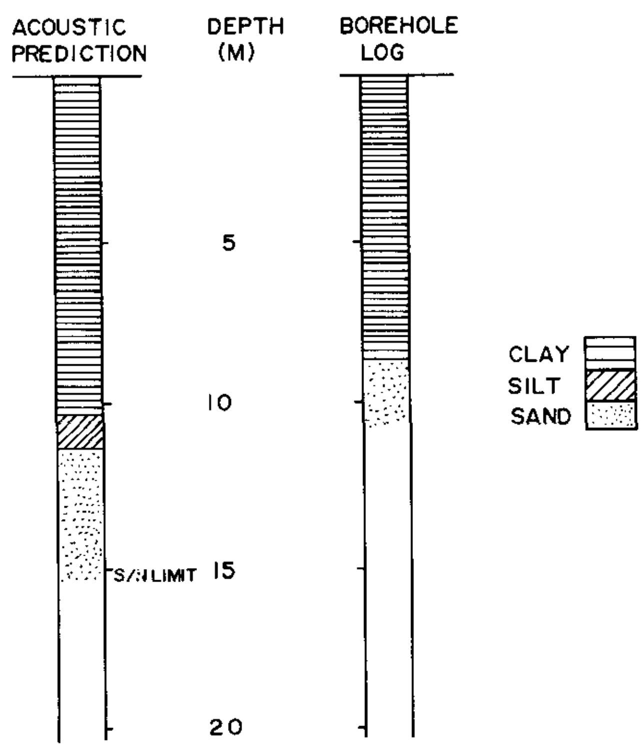

Fig. 7. Acoustic prediction versus borehole log.

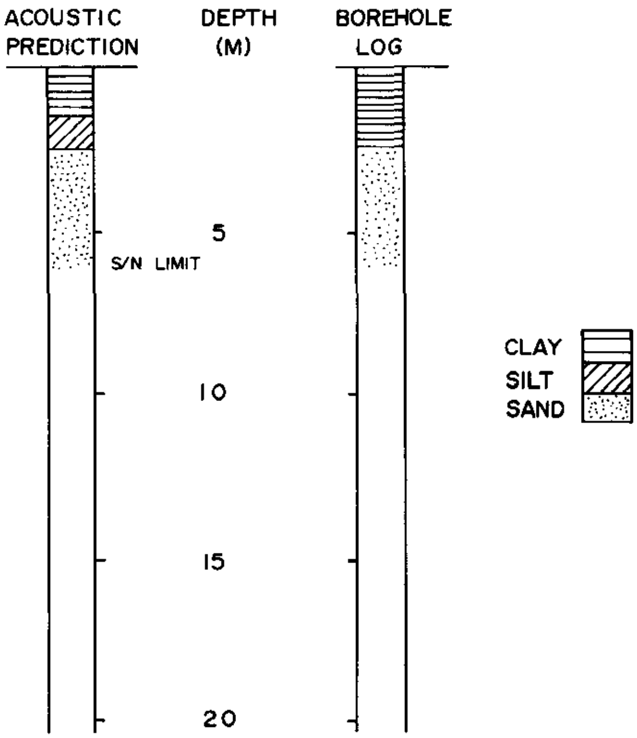

Fig. 8. Acoustic prediction versus borehole log.

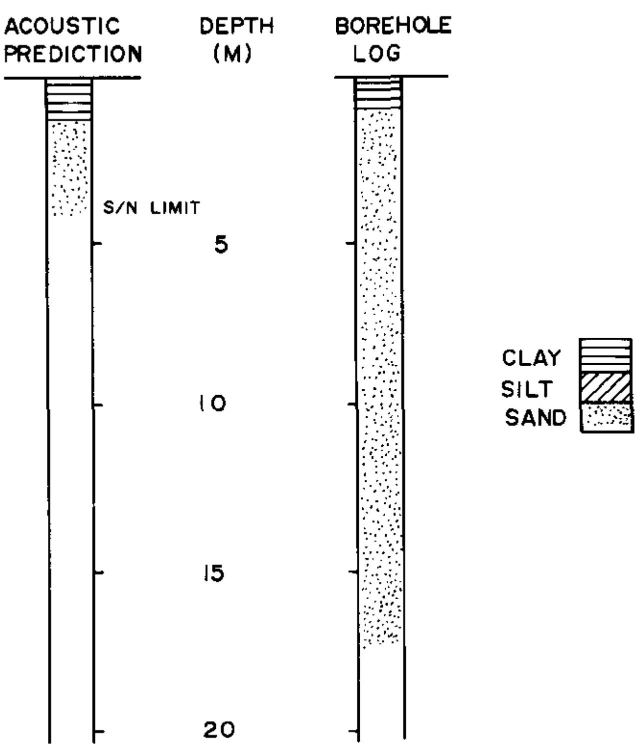

Fig. 9. Acoustic prediction versus borehole log.

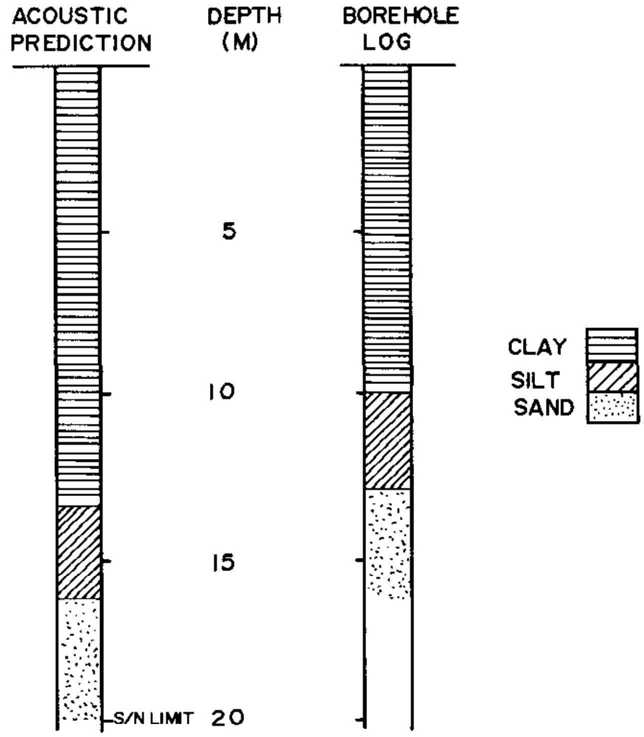

Fig. 10. Acoustic prediction versus borehole log.

Figure 11 is given to show the cross section of a river bed where the complete section was derived by computer from acoustic impedance rather than by manual interpretation of the seismic records. This figure points out the importance of utilizing digital processing for consistency in shallow seismic interpretation.

In all the data presented to date there were no significant multiple problems. If such do exist, classical seismic multiple suppression must be utilized before the data are processed.

The next step is the utilization of Hamilton's regression curves to predict geotechnical properties. The original data, Polar Gas Data, taken in 1975 were used with the model consolidated with ice data in the Beaufort Sea. The results for porosity predictions against actual core data are given in Figure 12. Likewise, the prediction of density versus actual core data is given in Figure 13.

Signal-to-Noise (S/N) Consideration

In all the previous figures there is a notation for S/N limit. It has been determined experimentally that confidence in the predicted results decreases for a signal-to-noise ratio below 5 db. The point where this level is reached is indicated as the signal-to-noise limit of confidence. It is interesting to note that the same phenomenon occurs in communication theory.

Conclusion

An approach to the estimation of the absorption and other losses in normal-incidence - high-frequency shal-

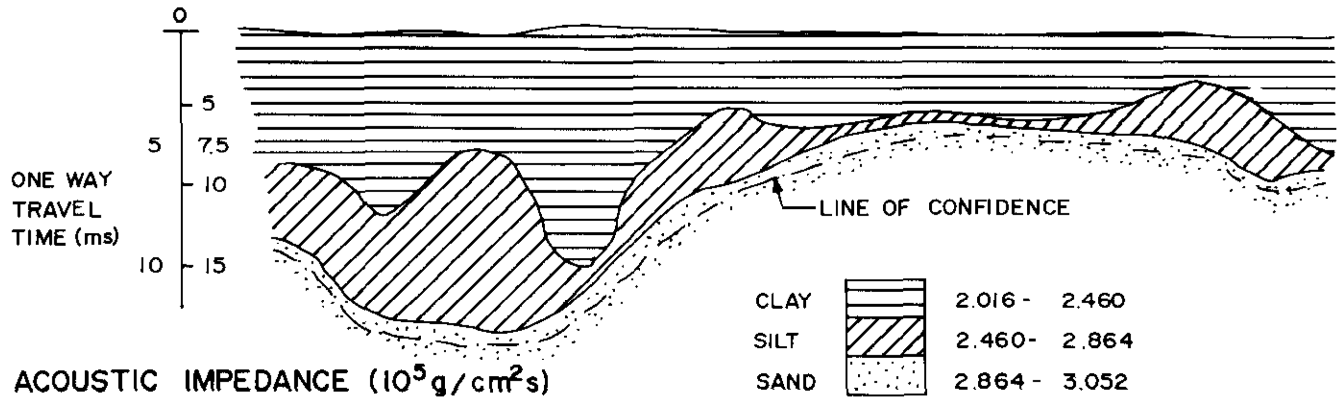

Fig. 11. Typical cross section of a river bed.

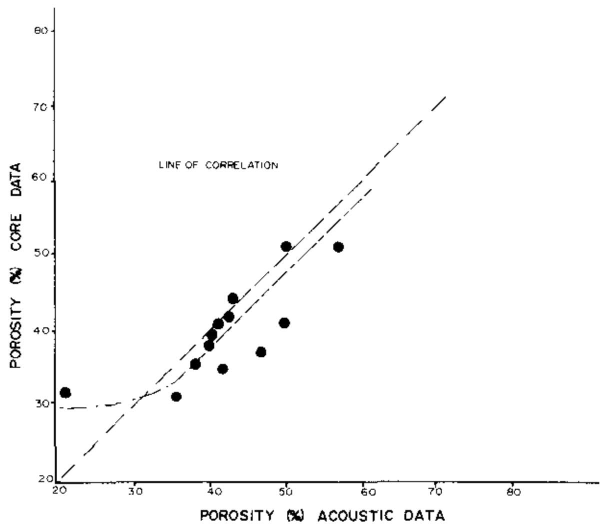

Fig. 12. Porosity versus porosity.

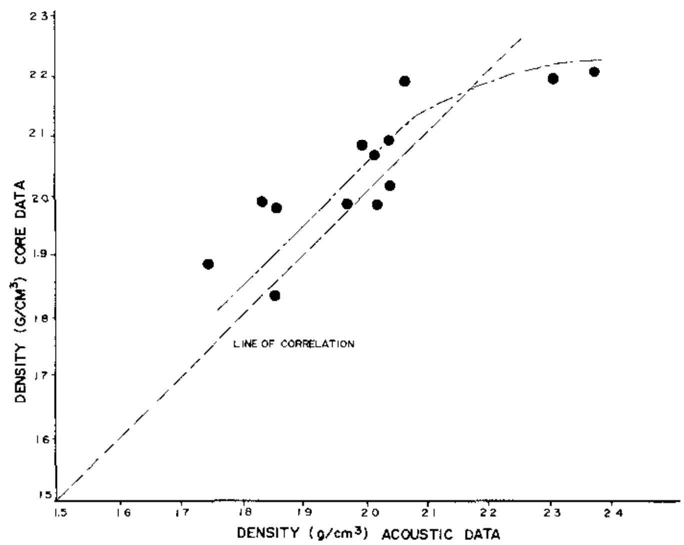

Fig. 13. Density versus density.

low seismics has been introduced. The model has shown itself a viable method for digital processing of the shallow seismic data, and the correlation for the prediction of general soils classification is good. Further work is needed to handle the problem of negative reflection coefficients, and to obtain a larger data base for the calibration approach. Both are under way.

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