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NOISE SUPPRESSION FOR DECONVOLUTION

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ABSTRACT

Deconvolution generally enhances seismic resolution, frequently at the expense of increasing noise content. Therefore, it is worthwhile to consider noise suppression as an essential step in deconvolution. Wiener filtering can be used for both optimum deconvolution and noise suppression. The combined Wiener noise suppression/deconvolution filter provides optimum performance in tests on both symmetric Ricker wavelets and minimum-phase wavelets. The noise suppression/deconvolution filter produces an unconstrained deconvolution for the case of zero noise and represents a prewhitened Wiener deconvolution filter for the case of random noise. A disadvantage of Wiener noise suppression/deconvolution is that the ratio of noise/signal power spectra is generally unknown and must be estimated. If this estimation is problematic, one can resort to the use of other noise suppression filters prior to deconvolution. One such filter is the output-energy filter which seeks to maximize the ratio of the filtered signal energy to the average noise power. Under the assumption of random noise, the output-energy filter can be simply obtained – by computing the eigenvector for the signal autocorrelation which is associated with the largest eigenvalue. In our tests with quasi-random noise examples, the output-energy filter becomes very similar to a running-average filter. Computational examples show that noise suppression filters can work to enhance deconvolution, provided we can make certain statistical assumptions about the noise.

INTRODUCTION

Deconvolution remains as one of the principal tools in the seismic processor's tool kit, especially for the purposes of multiple suppression and source wavelet compression (ref. Peacock and Treitel, 1969). Recent discussions have focused on the source wavelet deconvolution problem due to its resolution of thin beds in stratigraphic exploration. The May 1994 CSEG *Recorder* has an excellent discussion about the problems of wavelet phase estimation, with several different opinions about the validity of various seismic wavelet deconvolution methods. Although there is certainly not a consensus on how to estimate and remove the seismic wavelet, we address another part of the wavelet deconvolution problem, which is related to the signal-to-noise characteristics of deconvolved data. Our interest in this topic was aroused by the recent paper of Maklad et al. (1993).

The general nature of deconvolution is to flatten the amplitude spectrum, usually increasing the high-frequency content of the filtered output. This enhancement of high-frequency content in order to improve resolution of overlapping arrivals usually has the undesirable effect of decreasing the signal/noise ratio for the high frequencies. In fact, noise is increased within any frequency band where deconvolution is applied to signals which have fallen below the noise floor of the data acquisition system. Therefore, it is worthwhile to consider deconvolution approaches which increase resolution while maintaining a desirable signal/noise ratio in the data.

For these reasons, the paper of Maklad et al. (1993) is especially interesting. The paper uses a Wiener filtering approach to enhance signal to noise while deconvolving the data. If the ratio of signal-to-noise power can be accurately estimated over the complete frequency band, the Wiener noise suppression is optimum in a least squares sense. In other circumstances where we do not have reliable noise-to-signal power estimates, we may wish to apply alternative noise suppression methods. For these purposes, we focus on the research of Robinson and Treitel (1980), who dedicate a chapter of their book *Geophysical Signal Analysis* to a discussion of such noise suppression techniques.

In this paper, we compare the performance of noise suppression methods and wavelet deconvolution by using a number of synthetic examples based on a symmetric Ricker wavelet and a minimum-phase wavelet. This is done for cases of known and unknown signal-to-noise ratios.

DECONVOLUTION

In initiating our discussion, we consider the convolutional model for the seismic trace, $s(t)$, to be given by the convolution of a source wavelet, $w(t)$, with the reflectivity function or sequence of reflection coefficients, $r(t)$. Symbolically, we write:

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$$w(t)*r(t) = s(t), \tag{1}$$

where * denotes the process of convolution, as a shorthand notation for:

$$\sum_{\tau} w(\tau)\, r(t-\tau) = s(t). \tag{2}$$

In the frequency domain, the Fourier transform of the trace is given by the product of the Fourier transforms of the wavelet and the reflectivity function. Although the convolutional model for the seismic trace has itself been recently challenged by Ziolkowski (1991), this linear superposition model has proven to be a useful working hypothesis for many processing examples over several decades.

For the purposes of this discussion, we will assume that we have accurately estimated the wavelet time sequence, $w(t)$. Discussions on various types of wavelet estimation have been given by Ricker (1953), Robinson (1967a), Lines and Ulrych (1977), Robinson and Treitel (1980), Lines and Treitel (1985) and Ziolkowski (1991). Once the wavelet has been estimated, deconvolution can be obtained by computing a filter which shapes the wavelet to a spike or delta function. Robinson (1967b) gives a FORTRAN subroutine named SPIKE which successfully computes Wiener (least squares) spiking filters.

Ideally, in deconvolution we would design a filter, $f(t)$, to remove the effect of the wavelet by shaping it into a delta function, so that the ideal deconvolution output, $x(t)$, would be the geologically interesting reflectivity function. As pointed out by Treitel and Lines (1982), the ideal case would give:

$$x(t) = f(t)*w(t)*r(t) = r(t). \tag{3}$$

Although this idealized recovery of the reflectivity by deconvolution is generally not possible due to the band-limited and noisy nature of our data, it is the desired result.

In the time domain, we design a filter $f(t)$ such that

$$f(t)*w(t) = o(t), \tag{4}$$

where the output $o(t)$ (the resolving kernel) will hopefully resemble a zero-phase band-passed version of a delta function. Consequently, the desired deconvolution will be a band-passed version of the reflectivity function, $r(t)$.

In the frequency domain, this amounts to computing a filter whose Fourier transform is given by:

$$F(f) = 1/W(f), \tag{5}$$

where $W(f)$ is the Fourier transform of the source wavelet.

In terms of amplitude and phase spectra,

$$F(f) = |W(f)|^{-1}\, e^{-i\theta(f)}, \tag{6}$$

where $|W(f)|$ is the wavelet amplitude spectrum and $\theta(f)$ is the wavelet's phase spectrum.

Therefore, wavelet deconvolution can be viewed as a process of spectral whitening and wavelet dephasing. If the wavelet phase is believed to be zero-phase due to previous dephasing processing steps, the spectral whitening or division by the wavelet's amplitude spectrum is equivalent to deconvolution.

One of the usual drawbacks of deconvolution is the enhancement of high-frequency noise, due to the spectral whitening step. This can be recognized by examining the spectrum of $1/W(f)$. For the purposes of discussion, let us assume that the dephasing steps have been done so that we can consider $F(f)$ to be $1/|W(f)|$. The nature of the seismic wavelet usually requires that $F(f)$ has large values at the high frequencies. When $F(f)$ is multiplied by the trace spectrum, the deconvolution will tend to enhance the high frequencies at the expense of increasing the high-frequency noise in the trace.

One of the conventional methods of avoiding the problem of noise enhancement in deconvolution is to apply a prewhitened Wiener filter, which attempts to deconvolve the wavelet while suppressing the enhancement of filtered white noise. The design of these filters is described by Treitel and Lines (1982). The prewhitened Wiener filter design effectively adds a DC level to the wavelet's power spectrum to suppress the deleterious effects of noise enhancement in deconvolution. The method essentially assumes that the noise spectrum is white or distributed equally over all frequencies. The constrained (prewhitened) Wiener filter design solves an optimization problem in which resolution is maximized, subject to the constraint that filtered random noise power be limited.

However, if we have knowledge about the ratio of the noise-power spectrum to signal-power spectrum over an entire frequency band, then it is desirable to design a Wiener filter which handles the noise suppression filter in a least-squares optimum manner.

Noise Suppression with Wiener Filters

Wiener filters generally attempt to produce a filter which will produce a desired output when convolved with a particular input sequence. The filter is designed by using least squares to minimize the difference between filtered output and desired output.

In the case of noise suppression, we consider the recorded seismic trace, $y(t)$, to be given by the sum of signal, $s(t)$, plus noise, $n(t)$:

$$y(t) = s(t) + n(t). \tag{7}$$

The desired output of the noise suppression process is the signal, $s(t)$. Ideally, a noise suppression Wiener filter, $h(t)$, will produce $s(t)$ when convolved with $s(t) + n(t)$. A lucid formal derivation of such a filter is given by Lathi (1968). The frequency-domain representation for the Wiener filter, $H(f)$, is given by:

$$H(f) = S^*(f)\,S(f) \,/\, [S^*(f)\,S(f) + N^*(f)\,N(f)], \tag{8}$$

where $S(f)$ and $N(f)$ are the Fourier transforms of the signal and noise sequences and $N^*(f)$, $S^*(f)$ denote their complex conjugates. If we divide the numerator and denominator by the signal-power spectrum, $S^*(f)\,S(f)$, we obtain:

$$H(f) = 1.0 \,/\, [\,1 + NSR(f)], \tag{9}$$

where $NSR(f) = N^*(f)N(f)/S^*(f)S(f)$ is the ratio of the noise power to the signal power as a function of frequency.

A heuristic derivation for $H(f)$ is obtained by using a trick found in Claerbout (1976) for least-squares filters. First consider an ideal filter, $H(f)$, which produces signal from recorded data such that $H(f)\,[S(f) + N(f)] = S(f)$. Then multiply the equation by the complex conjugate of $S(f) + N(f)$ and use the assumption that the noise does not correlate with signal in order to eliminate crosspower terms and produce equation (8) or (9).

Now consider the frequency-domain representation for a filter, $G(f)$, which combines deconvolution with noise suppression. That is, find $G(f) = H(f)/W(f)$. We can write $G(f)$ as:

$$G(f) = 1/\{\, W(f)\,[1 + NSR(f)]\,\}. \tag{10}$$

This deconvolution filter, as discussed by Maklad et al. (1993) has some nice features which make it robust. For small noise-to-signal ratios where $NSR(f)$ approaches zero, $G(f)$ represents a spiking filter which computes the reciprocal of $W(f)$. For frequencies where noise-to-signal ratios are large, $G(f)$ will have a small value in order to suppress the amplitudes of filtered data in the noisy frequency bands. This robust deconvolution filter sequentially applies a Wiener noise suppression filter with a deconvolution filter in order to *selectively deconvolve* that part of the spectrum with good signal-to-noise ratios. In the special case of white random reflectivity and white random additive noise, this filter is equivalent to a prewhitened Wiener deconvolution filter. This can be seen by rearranging equation (10) as:

$$G(f)\,\{\,1 + NSR(f)\}\,W(f) = 1 \tag{11}$$

and recall that

$$NSR(f) = N^*(f)\,N(f) \,/\, S^*(f)S(f). \tag{12}$$

If noise is white, then $N^*(f)N(f) = \sigma_n^{\,2}$, where $\sigma_n^{\,2}$ is the noise power. If reflectivity is white then $S^*(f)S(f) = W^*(f)W(f)\,\sigma_r^{\,2}$, where $\sigma_r^{\,2}$ is the reflectivity power. Representing the ratio $\sigma_n^{\,2}/\sigma_r^{\,2}$ by $\sigma_{nr}^{\,2}$, we can rewrite equation (11) as:

$$G(f)\{\,1 + \sigma_{nr}^{\,2}/W^*(f)W(f)\}\,W(f) = 1. \tag{13}$$

If we now multiply by $W^*(f)$, we obtain the expression:

$$\{\,W^*(f)W(f) + \sigma_{nr}^{\,2}\}\,G(f) = W^*(f). \tag{14}$$

This is the frequency-domain representation of the normal equations for a Wiener spiking filter in which $w(t)$ is the input wavelet and the desired output is a spike, $\delta(t)$. Since the power spectrum of the wavelet has a DC value (prewhitened component) added to it, equation (14) produces the solution for a *prewhitened* Wiener filter. Therefore, for the special case of white additive noise and a white reflectivity sequence, the combined noise suppression/deconvolution filter is equivalent to a *prewhitened* Wiener filter.

The nontrivial problem in the Wiener noise suppression filter design is the *estimation* of $NSR(f)$, the ratio of the noise-power spectrum/signal-power spectrum. Unfortunately, our recordings are of the summation of the noise and signal and their complete separation is impossible. We must rely on statistical differences between noise and signal to provide the best estimation of the noise/signal power. If we make repeated measurements of noisy recordings, we can use the fact that the signal is redundant and the noise will tend to be unpredictable. One method of estimating the noise-to-signal power uses crosspower and autopower spectra for traces containing a common signal. If we have traces x_1 and x_2 with the same signal and different uncorrelated noise realizations, the computation of $H(f)$ can be estimated by computation of $P_{12}(f)/P_{11}(f)$. To see that this gives an approximation of $H(f)$, we write out these spectra in terms of signal and noise.

$$P_{12}(f)/P_{11}(f) = \{[S^*(f) + N_1^{\,*}(f)][S(f) + N_2(f)]\}$$
$$/\,\{[S^*(f) + N_1^{\,*}(f)][S(f) + N_1(f)]\}. \tag{15}$$

If noise is uncorrelated, then all crosspower terms involving $N_1(f)$ and $N_2(f)$ become vanishingly small and we have

$$P_{12}(f)/P_{11}(f) \approx S^*(f)S(f)/\{S^*(f)S(f) + N_1^{\,*}(f)N_1(f)\}. \tag{16}$$

This is often a valid approximation to the expression for $H(f)$ in (9). For this approximation method, we could also use $P_{21}(f)$ or $0.5[(P_{12}(f) + P_{21}(f)]$ in the numerator to obtain estimates of $H(f)$. Our results suggest that there is not a great deal of difference in these various approximations. Unfortunately, the assumptions used in these approximations for $H(f)$ are often violated with real data since two traces rarely have exactly the same signal, and some noise may be correlated with signal.

Output-Energy Filters

The Wiener noise suppression filter's success is dependent on our ability to estimate the ratios of the noise power to the signal power. An alternative noise suppression filter is the output-energy filter as described by Robinson and Treitel (1980). The output-energy filter seeks to maximize the ratio of (filtered signal energy)/(average noise power). The filter computation requires the solution of a generalized eigenvalue problem in which the autocorrelation of the signal and the autocorrelation of the noise must be known. In the case of white noise, the noise autocorrelation matrix is the identity matrix and the output-energy filter is a solution of the eigenvector equation:

$$\mathbf{R}\,\mathbf{a} = \lambda_{max}\,\mathbf{a}\,,$$

where $\mathbf{R}$ is the autocorrelation matrix of the signal, λ_{max} is the maximum eigenvalue of the autocorrelation matrix and $\mathbf{a}$ is the associated eigenvector. Since the autocorrelation matrix is a symmetric Toeplitz matrix, the output-energy filter $\mathbf{a}$ is either symmetric or antisymmetric (Robinson and Treitel, 1980). In general, this computation of the output-energy filter is fast and straightforward, provided we have an accurate estimate of the signal autocorrelation and we have some idea what the length of the filter should be. Based on our experiments with synthetic data, we have found that it is generally effective to use short filter lengths, and we have found that the output-energy filter closely approximates a running-average filter. It is not obvious why the output-energy filter should behave in this way, other than the fact that a running average can be effective for random noise reduction prior to deconvolution.

Numerical Comparisons

In comparing the characteristics of deconvolution with noise suppression, we examine a number of noisy synthetic examples. Filter performance is compared for various wavelets, reflectivity sequences and noise realizations.

Figures 1a-h show filter performance for a zero-phase symmetric Ricker wavelet (with peak frequency of 30 Hz). Ideally, the filtered output of wavelet deconvolution will resemble a band-passed spike or delta function. Program SPIKE from Robinson (1967b) computes the deconvolution based on an optimum spike position so that the deconvolution results should be shifted accordingly with the optimum spike position. With a symmetric wavelet of Figure 1a, the addition of noise can significantly affect the spiking position. Even for the case of no additive noise, the Wiener spiking filter can produce undesirable high-frequency side lobes in the filter output as shown in Figure 1b. As indicated by Treitel and Lines (1982), a small amount of prewhitening in the Wiener deconvolution filter can reduce these high-frequency side lobes at the expense of a broader filter output. Figure 1c illustrates this effect for the case of a Wiener spiking filter with 1% prewhitening. However, a noisy version of this wavelet in Figure 1d with a signal-to-noise ratio of 5, produces problems for both unconstrained and constrained (prewhitened) Wiener deconvolution filters. The filtered outputs of Figure 1e and 1f for both the unconstrained and constrained spiking filters show an output which is littered with high-frequency noise.

In this noisy case, it is better to apply the combined Wiener noise suppression filter with deconvolution, as suggested by Maklad et al. (1993). The output of the noise suppression filter applied to the wavelet produces the wavelet shown in Figure 1g. If this wavelet is deconvolved, the output shown in Figure 1h has less noise than the other deconvolutions.

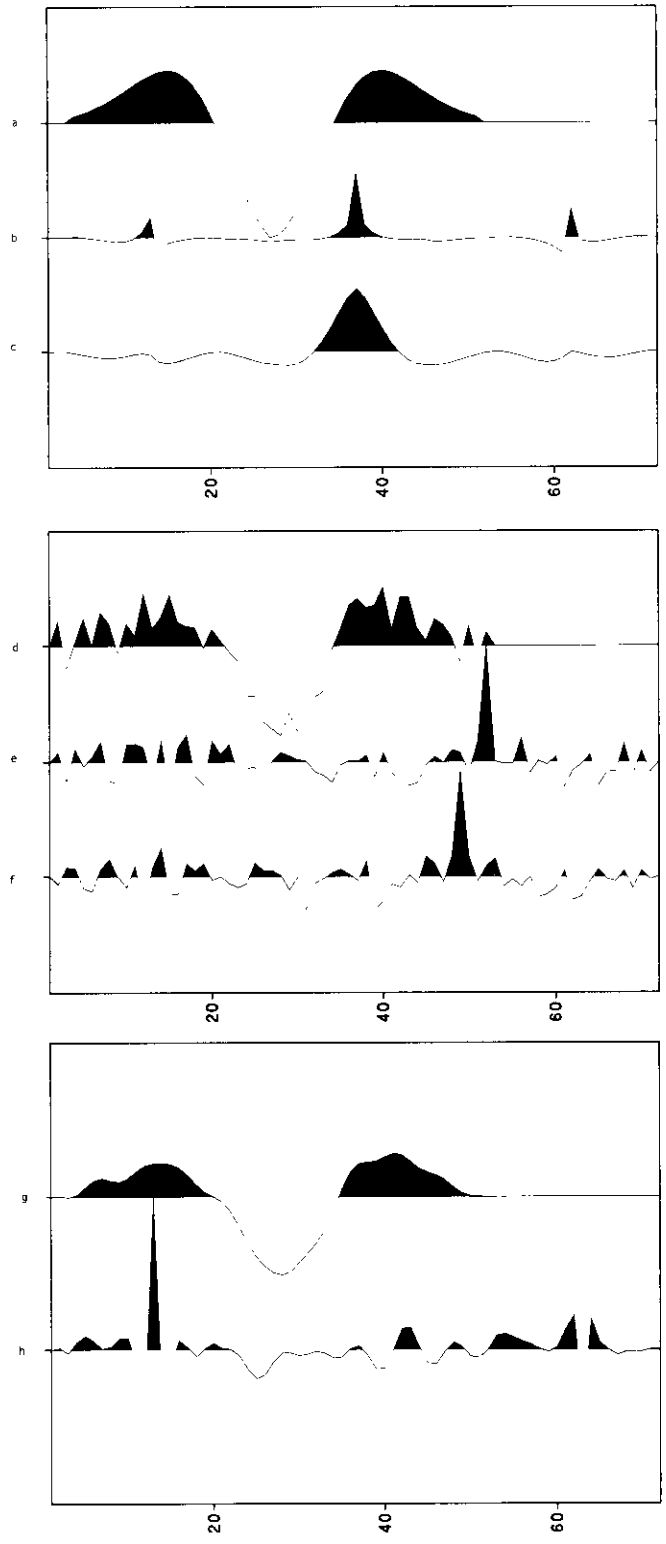

Fig. 1. (a) Input Ricker wavelet used in tests with peak frequency of 30 Hz. (b) Output of Wiener filter designed to spike the Ricker wavelet of Figure 1a. (c) Output of constrained Wiener spiking filter with a prewhitening level of 1%. The horizontal scale for this and all of the following examples denotes sample number. (d) Noisy version of the input Ricker wavelet produced by adding pseudo random numbers to the wavelet of Figure 1a. (e) Output of unconstrained Wiener spiking filter for the noisy input wavelet of Figure 1d. (f) Output of constrained Wiener spiking filter for the noisy input wavelet using a prewhitening level of 1%. (g) Output of Wiener noise suppression filter applied to the noisy wavelet of Figure 1d. (h) Output of Wiener spiking filter to the wavelet of Figure 1g.

Similar results hold true for deconvolution of a minimum-phase wavelet used by Treitel and Lines (1982), which is shown in Figure 2a. This damped sinusoidal wavelet has the nice property that it can be exactly deconvolved with a causal spiking filter of length 3, so it is not surprising that the numerical solutions produce an excellent spike, as shown in Figure 2b. However, the addition of noise to produce the noisy wavelet of Figure 2c creates disturbing effects on the deconvolution, as shown by Figure 2d. It is seen in Figure 2e that prewhitening does not hold a panacea for this noisy deconvolution, either. Application of the Wiener noise suppression filter again effectively suppresses the noise in the wavelet as shown in Figure 2f. The subsequent deconvolution of this wavelet, as shown by Figure 2g, is a marginal improvement over the previous methods. The combined Wiener noise suppression/deconvolution filter does well with these synthetic tests.

However, we should repeat our caveat concerning the use of Wiener noise suppression filters. Their effective use is based on accurate estimates of *NSR(f)*, the ratio of the noise-power spectrum to the signal-power spectrum over a full band of frequencies. This is a somewhat unrealistic assumption and we will need to now reexamine our estimation strategies for *NSR(f)* for a more realistic seismic trace example.

Figure 3a shows a reflectivity response obtained from sonic and density logs from an offshore Newfoundland well. The wavelet of Figure 2a is convolved with this reflectivity response and then quasi-random noise is added to produce the synthetic seismic trace of Figure 3b.

In order to see whether we can achieve better results with noise suppression filtering prior to deconvolution, we compare noise suppression capabilities of the various filtering approaches. Figure 3c shows a comparison of the filtered traces for Wiener filtering with known *NSR(f)*, Wiener filtering with two estimations of *NSR(f)*, a summation of traces, and the result of output-energy filtering. For these synthetic examples, a comparison of filtering results can be done quantitatively for the various methods by computing the sum of absolute values of differences between filtered output and signal. These results are summarized in Table 1. For this table the trace amplitudes are scaled to have a maximum amplitude value of 1.0.

Table 1. Comparison of noise suppression methods.

Method	Average Absolute Error
Signal + noise (no filtering)	.0066
Wiener filter (ideal)	.0037
Wiener filter [est. with $0.5(P_{12} + P_{21})$]	.0071
Wiener filter (est. with P_{12})	.0090
Stack (ideal)	.0043
Output-energy filter	.0049

The best performance for noise suppression was obtained by Wiener filtering with known *NSR(f)* values. Since this is usually an unrealistic assumption, we examined the crosspower estimation methods and found these methods were not as reliable as stacking or the use of an output-energy filter. An output-energy filter of length 5 was used and, essentially,

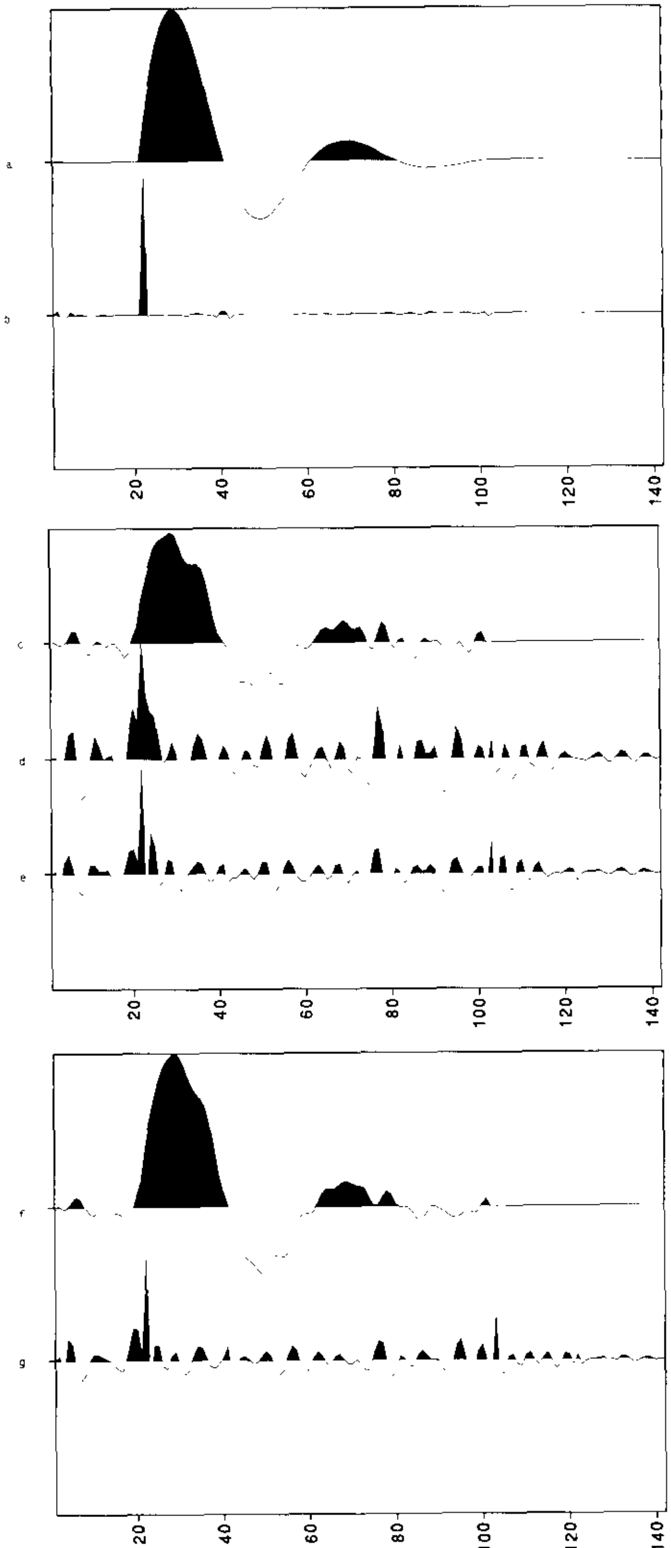

Fig. 2. (a) A minimum-phase wavelet used by Treitel and Lines (1982). This wavelet is obtained by applying exponential damping to a sinusoid of frequency 20 Hz. It has an exact spiking filter of length 3. (b) Output of unconstrained Wiener filter of length 40 which is applied to the wavelet of Figure 2a. (c) Noisy wavelet obtained by adding random numbers to the wavelet of Figure 2a. (signal/noise ratio = 5). (d) Output of unconstrained deconvolution applied to the noisy wavelet of Figure 2c. (e) Output of constrained Wiener deconvolution with a prewhitening level of 1%. (f) Output of Wiener noise suppression filter when applied to the noisy wavelet of Figure 2c. (g) Output of Wiener deconvolution on the wavelet of Figure 2f.

became a running-average filter which worked effectively but not quite as effectively as the ideal Wiener noise suppression filter. The stacking of traces with identical signals did slightly better than the output-energy filter and slightly worse than ideal Wiener noise suppression. The stacking was ideal in the sense that the summed traces contained *identical* signals with different random noise realizations.

This comparison of noise suppression methods is somewhat less detailed than Chapter 14 of Robinson and Treitel (1980), which considered matched filters and output-energy filters. Matched filters are particularly advantageous in Vibroseis and marine chirp methods, but they require accurate knowledge of the signal's shape, whereas output-energy filters require only accurate estimates of the signal autocorrelation (an easier demand).

Our conclusions were similar to those of Robinson and Treitel (1980) in that the choice of noise suppression methods essentially depends on the reliability of the noise and signal information available. From the viewpoint of ease of design, stacking and output-energy filters were desirable despite the fact that they were slightly less effective than the ideal Wiener noise suppression filter.

A comparison of deconvolution after noise suppression in Figure 3d gave a favourable rating to the use of output-energy filters prior to deconvolution. A comparison of the absolute differences between the deconvolution and the actual reflectivity function is given in Table 2. In all cases, spiking deconvolution filters with a length of 40 and 1% prewhitening were applied to the noisy traces of Figure 3c. (All traces were normalized to the same maximum amplitude.)

Table 2. Comparison of deconvolutions after noise suppression.

Method of Noise Suppression	**Average Absolute Error**
No noise suppression	
(except prewhitening during decon)	16.6
Ideal Wiener noise suppression	16.7
Wiener noise suppression	
(using P_{12}/P_{11})	15.9
Wiener noise suppression	
(using 0.5 (P_{12} + P_{21})	16.4
Stacked data	17.1
Output-energy filter	13.4

All noise suppression/deconvolution approaches were about the same in performance except for the best case in which the output-energy filter was used prior to deconvolution. The output-energy filter's effect of slightly smoothing the data prior to deconvolution resulted in a deconvolution which was about 20-25% better than the other methods. The reason for this may lie in the fact that the output-energy suppressed the high-frequency spectrum of the noise, which is the noise contribution most damaging to deconvolution.

Conclusions

This study of noise suppression and deconvolution caused us to reexamine the usual techniques of constrained (prewhitened) Wiener spiking filters and to compare this

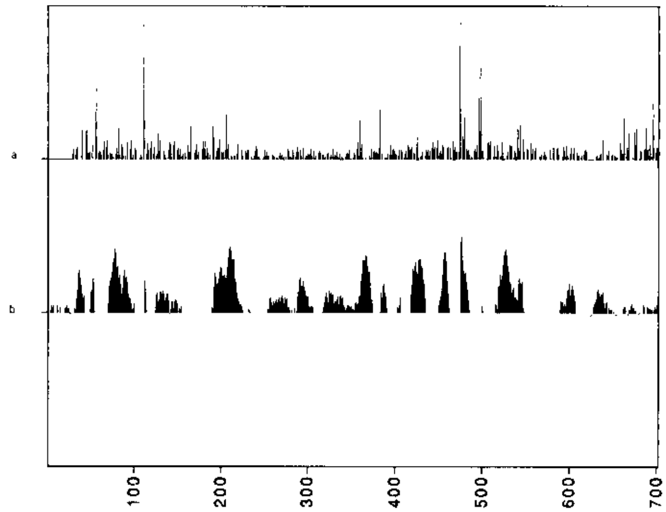

Fig. 3. (a) Reflectivity response obtained using the sonic and density logs from an offshore Newfoundland well. (b) Synthetic seismic trace obtained by convolving the wavelet of Figure 2a with the reflectivity response of Figure 3a.

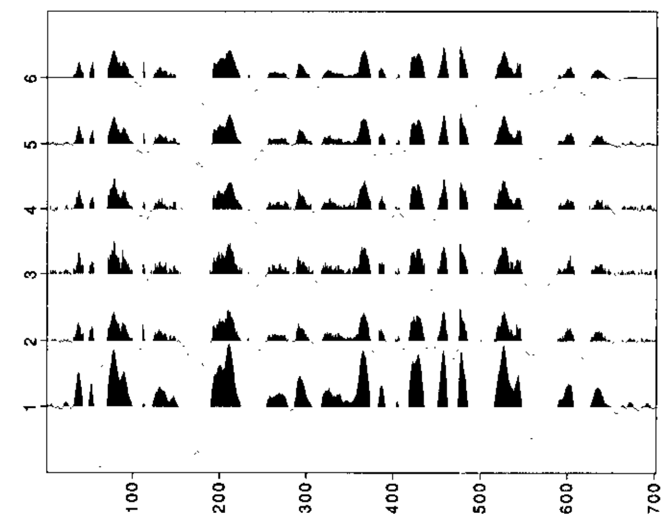

Fig. 3. Cont'd (c) Signal trace compared to traces that have undergone noise suppression. The traces as numbered (from top to bottom) are: 6. signal trace (desired output of filters); 5. ideal Wiener noise suppression; 4. Wiener noise suppression using 0.5 [$P_{12}(f)$ + $P_{21}(f)]/P_{11}(f)$; 3. Wiener noise suppression using $P_{12}(f)/P_{11}(f)$; 2. stacked data; 1. output-energy filtered trace.

method with the methods of noise suppression filtering prior to deconvolution. In particular, the Wiener noise suppression/deconvolution filtering and output-energy filter/deconvolution methods appear promising. The critical step in use of the former method is the estimation of signal-to-noise ratios. If these estimations are not accurate, it may prove advisable to use the simpler techniques of prewhitened Wiener deconvolution or to apply output-energy filtering prior to deconvolution. In particular, the output energy noise suppression filter was easy to compute and, when combined with spiking deconvolutions, it proved to be the most effective on the synthetic data which we tested.

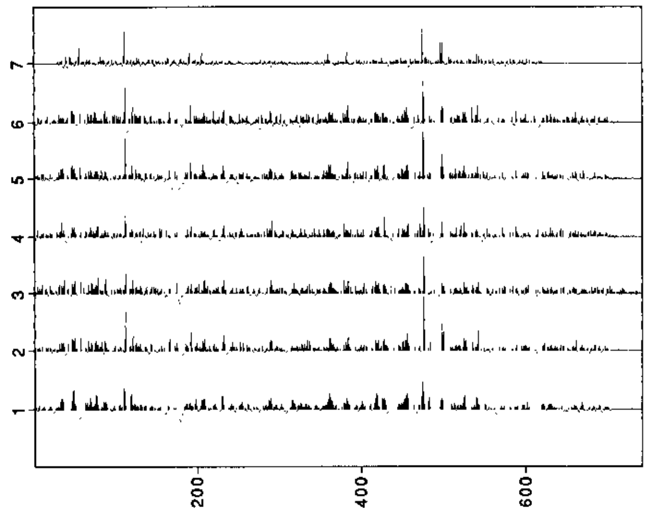

Fig. 3. Cont'd. (d) Comparison of prewhitened deconvolutions after various noise suppression filters. These traces as numbered (from top to bottom) are: 7. reflectivity function (desired output of deconvolution); traces 6-1 are deconvolutions of traces following: 6. no noise suppression; 5. ideal Wiener noise suppression; 4. Wiener noise suppression using 0.5 $[P_{12}(f) + P_{21}(f)]/P_{11}(f)$; 3. Wiener noise suppression using $P_{12}(f)/P_{11}(f)$; 2. stacking; 1. output-energy filtering.

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