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NMO DISTORTION

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ABSTRACT

A complete classification system for distortion produced by second-order NMO application is developed. Inequalities which produce the distortion classification as a function of time for any given NMO function and sample interval are presented. Equations are developed which give the smallest offset at which a given amount of distortion occurs for each class of distortion. A new type of display which permits easy visualization and comparison of distortion class for one or more NMO functions is presented.

INTRODUCTORY REVIEW OF SOME NMO AND SEISMIC VELOCITY BASICS

Most data interpreted by geophysicists consist of stacked traces. Before stacking, each trace in the stacked gather is corrected (by time shifting sample or interpolated values) to produce an approximation of a zero-offset trace by applying the standard NMO equation:

$$t(x) = (x^2/v^2 + t^2(0))^{1/2} \quad (1)$$

where:

- x is the source to receiver distance (also called the offset);
- $t(0)$ is the event arrival time for an offset of zero; and
- v is a parameter with consistent units of velocity.

This equation is derived using the geometric properties of right angle triangles to find the arrival time of a reflected ray in a single layer subsurface having zero dip and a constant isotropic velocity v. The actual NMO (normal moveout) correction applied at offset x and for time $t(0)$ is a time shift of size $t(x) - t(0)$.

Despite the extraordinary simplicity of the model from which it is derived, the NMO equation is routinely used (with adjustments to the velocity parameter) in exploration to correct data recorded in areas with complex geology. A brief justification for this follows.

A model with a single dipping layer will have an NMO equation of the same form as equation (1). In this case, the v term must be replaced by $v/\cos(\theta)$ where theta is the geologic dip (not the apparent time section dip). Here, it is implicitly assumed that the offset x is restricted so that the entire travel path of the ray falls within the triangular wedge formed by the dipping boundary. Note that because the cosine of ten degrees is about 0.98, the size of the NMO correction does not change very much for small dips. This makes the NMO equation, even without dip correction, a fairly robust approximation for the purposes of making a dynamic correction for offset. Provided that the proper (dip adjusted) velocity parameter is used, the correction is also robust for linear dipping reflections in the single layer case.

For a subsurface that consists of N homogeneous zero dip boundaries, it can be shown that the exact equation for corrections approaches the standard NMO equation as x approaches zero so long as the velocity term v is replaced by the root-mean-square velocity (rms velocity, defined below) of the event being corrected.

The rms velocity associated with an arrival time is an arbitrary mathematically defined quantity that is related to the model layer velocities (and, implicitly, a ray-path) as follows:

$$V_i = \left(\sum t_i v_i^2 / \sum t_i\right)^{1/2}. \quad (2)$$

Strictly speaking, this quantity has a meaning only with respect to a specific ray, since in this definition t_i is the travel-time in layer i and this time depends on the raypath which is different for each offset.

For the purpose of stacking data from flat layered models, the raypath is implicitly chosen to be that of a vertical ray. For this reason, we may think of equation (2) as defining the rms velocity of the model or, equivalently, of the model record. For dipping linear layers, the relevant ray for each event is the normal incidence zero-offset ray.

From experience, we know that if some type of statistical velocity analysis such as a velocity spectrum is done on a model record, the observed velocities are, for most realistic models, within about ten percent of the rms velocity as defined above. In certain cases such as those with high-velocity surface layers (e.g., permafrost) or with very low

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velocity layers, the rms equation for a zero-offset ray may no longer relate in any simple way to the observed stacking velocity derived from a statistical method.

Even greater errors may occur in such cases if the rms equation is rearranged in order to deduce interval velocities from the observed velocity analysis. In such cases, there is no longer any simple useful relationship between the layer velocities, the arbitrarily defined rms velocity of a zero-offset ray and a statistically determined stacking velocity (for example, from a velocity spectrum). Additional difficulties occur if boundaries are curved, if near-surface statics are significant, if the layer velocities are anisotropic, or when some of the energy arrives from out of the plane of the section.

The appropriateness of the velocity corrections applied to traces before stacking is probably the most significant single determinant of the final quality of seismic sections. This fact makes the control of errors and distortion that may occur in the NMO step critically important.

For the rest of this paper, we will assume that the standard NMO equation presented above, with a suitable velocity parameter, is being used to correct seismic traces and we will be concerned only with problems, such as distortion, that arise due to its fundamental properties. A similar analysis may be conducted for cases that require a modification (such as a shift of the origin of the coordinate system) of the standard NMO equation or for other low-order nonlinear equations than might be used to make dynamic corrections.

Outline of Analysis

1) Asymptotic behaviour of the NMO equation will be pointed out as an aid for visualization of NMO curve behaviour at far offsets and for use in NMO distortion classification.
2) Two major classes of NMO distortion that can be controlled with offset-constrained NMO will be presented.
3) Equations will be presented which give the minimum offset at which a specified amount of distortion occurs for each of the distortion subclasses.
4) A complete set of subclasses for NMO distortion will be described so that it is clear how software may be written to constrain the amount of distortion that occurs when NMO is applied.

Asymptotic Behaviour

The equation $t(x) = (x^2/v^2 + t^2(0))^{1/2}$ is an expression in Cartesian geometry for one of the standard conic sections, namely the hyperbola. It has been known since the time of the ancient Greeks that this curve is asymptotic to a straight line. This result is a trivial one if we examine the behaviour of the ratio $(x^2/v^2)^{1/2} / (x^2/v^2 + t^2(0))^{1/2}$. This ratio obviously approaches 1 as x approaches infinity since it is equivalent to the equation $(1/v^2) / (1/v^2 + t^2(0)/x^2)$ in which $t(0)$ and v are constants with respect to x.

It is also obvious that the numerator of the ratio reduces to x/v which represents a straight line with slope $1/v$ through the origin of the coordinate system.

Slowness

It is often convenient to use the slowness parameter s rather than the velocity parameter v.

Using the definition $s = 1/v$, the standard NMO equation may be written:

$$t(x) = (s^2x^2 + t^2(0))^{1/2} \tag{3}$$

and the equation of the asymptote is:

$$t(x) = sx\ . \tag{4}$$

It is clear that the slowness parameter s is also the slope of the hyperbola's asymptote.

Using asymptotes to classify far-offset distortion

Type 1 – If $t_1(0)$ and $t_2(0)$ are two different arrival times which have associated velocity parameters v_1 and v_2, respectively, then if $v_1 = v_2$, we know that the standard NMO curves for both arrival times are asymptotic to the same line having slope $s = 1/v_1 = 1/v_2$. This immediately tells us something significant, at least for far offsets, about NMO distortion for the seismic data between the two arrival times, namely, that the time between the curves after NMO correction will be greater than before correction. This is commonly referred to as NMO stretch, and it is now clear that we have found one situation in which this particular type of distortion will occur, namely at far offsets when event pair velocities are equal (see Figure 1 for an example).

Type 2 – If $t_1(0)$ and $t_2(0)$ are two different arrival times which have associated velocity parameters v_1 and v_2, respectively, then if $v_1 < v_2$ and $t_1(0) < t_2(0)$, we know that the standard NMO curves for the arrival times are asymptotic to two different lines having slopes s_1 and s_2, respectively, and that $s_1 > s_2$. It is clear that since the second curve starts (at zero offset) at a later time than the first, but becomes asymptotic to the line with the smaller slope, that the two curves must cross at some offset (see Figure 1 for an example).

Near, and inside the crossing point, the curves will converge. Near, and beyond the crossing point, the curves will diverge. If NMO is applied beyond the offset at which crossover occurs, the result will be a time reversal which we presume should always be avoided. We know from geologic principles and from observations in well logs that layer velocities tend to increase with depth. Since the layer velocities play a major role in determining the NMO velocity parameter, we also observe that the NMO velocity parameter also tends to increase with increasing arrival time. For this reason, we should expect that Type 2 distortion will be the most common potential problem (as well as being one of the two, as we will see, most serious).

Fortunately, recording offsets are often restricted such that much of the seismic record may be corrected using the standard NMO equation without catastrophic errors (such as

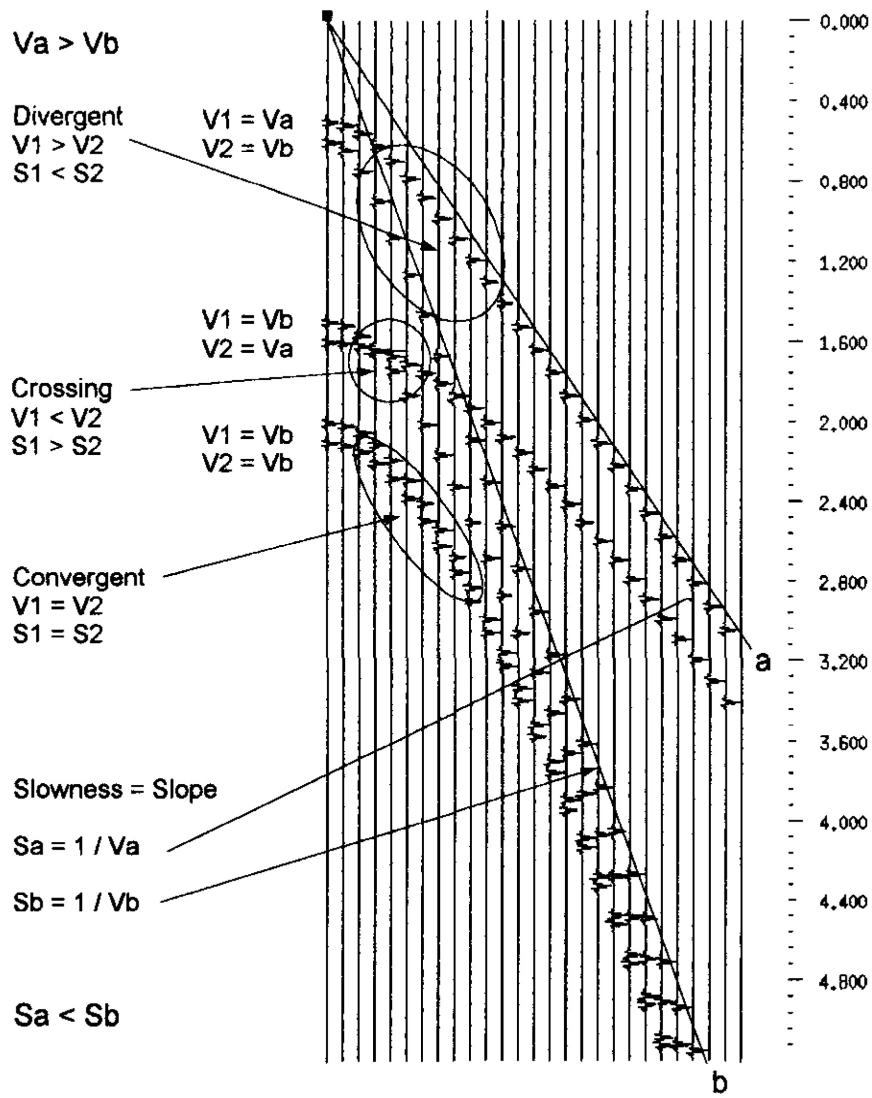

Fig. 1. Asymptotic behaviour.

time reversals) even if the NMO program does not restrict itself to making corrections only for properly restricted offsets. As we show below, there is no need to rely on good fortune since the offset at which crossover occurs is easy to determine and to program as a constraint into NMO-correction software.

Type 3 – The only remaining case (for far offsets) is similar to Type 2 except that $v_1 > v_2$. In this case, the two NMO curves will diverge (without time reversal) at far offsets. The NMO distortion will be a compression for far offsets (see Figure 1 for an example).

Distortion Classes

We will distinguish two major classes of NMO distortion, namely, global and local which arise from fundamental nonlinear properties of the standard NMO equation.

Global (or catastrophic) distortion

This type of distortion will occur if trace data from offsets equal to or greater than the crossover distance is NMO corrected. We know from the discussion above concerning asymptotes that this problem is associated with the condition $v_1 < v_2$.

The results of this type of distortion include:

(a) time reversal of wavelets;

(b) singular (crossover) points at which the distortion magnitude is numerically infinite;

(c) multivalued mapping in which energy that should be stacked at some specific time is also moved to, and incorrectly contributes to the energy which is properly corrected to and stacked at, some other time.

See Figure 2 which illustrates the effects of global distortion.

The term global distortion is particularly suitable for cases in which $v_1 < v_2$ and the event pair times are "considerably different", that is, much greater than the smallest time interval for which the seismic wavelet can resolve two different events. As an example, suppose that the energy for a Tertiary event should be stacked at a time of 1.0 seconds and the energy for a Mississippian event should be stacked at

1.8 seconds. If the stacking velocity of the deeper event is greater, as is most likely the case, and if long enough offsets occur in the traces that will be NMO corrected and stacked, then the two events will cross. At the crossing point, if *unconstrained* NMO is used, energy from both events will be moved to both event times.

If one of these events is significantly stronger, the damage that results is worse for the weaker event. Beyond the crossover offset, energy from other events will be similarly moved to more than one zero-offset time and, subsequently, stacked at both the correct and at one or more incorrect times. In geology, one avoids mixing samples from the Mississippian with those from the Tertiary. In geophysics, one should be at least as careful to avoid analogous errors that may occur when applying NMO.

Local distortion

Local distortion is defined as the distortion that is calculated with respect to an event pair that has a "small" difference in zero-offset arrival time. We will in fact define local distortion to be the calculated distortion for an event pair that has a time separation of exactly the seismic trace sample interval. This is suitable since this interval is smaller than the minimum time interval that a real seismic wavelet can resolve. It also provides a numerical value for distortion that is useful to provide a practical constraint for NMO offsets in programs which typically interpolate between trace samples in order to obtain an event amplitude to be time shifted to the event's zero-offset time.

Next, an equation for the crossover offset will be presented. Then, we present a complete classification system for NMO distortion for both near and far offsets and equations that give the maximum suitable offset for a specific selected amount of distortion.

The crossover offset

By equating the standard NMO equations for two events with different zero-offset times T_1, T_2, we can solve for the offset x of crossover using elementary algebra provided that the conditions $T_1 < T_2$ and $v_1 < v_2$ (i.e., $s_1 > s_2$) hold. The result is:

$$x = ((T_2^2 - T_1^2) / (s_1^2 - s_2^2))^{1/2}, \quad T_1 < T_2, s_1 > s_2 . \tag{5}$$

It is obvious from the symmetry of hyperbolas about the time axis that both $+x$ and $-x$ are solutions for cases in which signed offset distances are used. For those using this equation in software, the standard techniques to avoid loss of numerical precision for expressions like this should be observed.

The General Offset for a Specific Distortion

Table 1 provides a classification of distortion for far offsets. This system is based on asymptotic behaviour and is valid because two NMO curves must either converge or diverge monotonically after a certain offset is reached. As it turns out, for near offsets, two NMO curves may converge over some range of offsets before they begin to diverge. This causes the classification of distortion and the calculation of the offset for a given distortion to be slightly more complex for the near-offset region, as we will see in this section.

Table 1. Asymptotic (far offset) event pair behaviour.

Condition	Behaviour	Distortion	Catastrophe
$v_1 < v_2, s_1 > s_2$	Cross	Compression	Time Reversal, singularity
$v_1 = v_2, s_1 = s_2$	Converge	Stretch	None
$v_1 > v_2, s_1 < s_2$	Diverge	Compression	None

Magnitude of distortion defined

Stretch Case – Here the time difference for two events is larger after NMO is applied.

$$\text{Percent distortion: } Ps = 100\,((\Delta t(x) - \Delta t(0)) / \Delta t(0)). \tag{6}$$

Compression Case – Here the time difference for two events is smaller after NMO is applied.

$$\text{Percent distortion: } Pc = 100\,((\Delta t(0) - \Delta t(x)) / \Delta t(0)) \tag{7}$$

or

$$-Pc = 100\,((\Delta t(x) - \Delta t(0)) / \Delta t(0)). \tag{8}$$

Here we define $\Delta t(0) = t_2(0) - t_1(0)$ and $\Delta t(x) = t_2(x) - t_1(x)$.

The ratio $P = Ps/100$, for the stretch case, or $P = Pc/100$ for the compression case, will also be useful notation in the following development of the offset equation.

We now summarize a method for finding the general equation for offset x with a specified distortion ratio P.

In the stretch case (equation 6), we have a specified Ps and calculate $P = Ps/100$. Substitution into equation (6) and rearranging gives:

$$\begin{aligned}(P + 1)\,\Delta t(0) &= \Delta t(x) \\ &= t_2(x) - t_1(x) \\ &= (s_2^2x^2 + t_2^2(0))^{1/2} - (s_1^2x^2 + t_1^2(0))^{1/2} .\end{aligned}$$

Now make the convenient substitutions:

$y = x^2$;
$T_i = t_i^2(0)$;
$\sigma_i = s_i^2$; and
$\beta = (P + 1)\,\Delta t(0)$, which is a constant with respect to x.

We now have:

$$\beta = (\sigma_2 y + T_2)^{1/2} - (\sigma_1 y + T_1)^{1/2}, \tag{9}$$

which we must solve for y.

Note that since the NMO hyperbola is symmetric about the time axis, if x is a positive offset at which a specific amount of distortion occurs, then the same amount of distortion will also occur at an offset of $-x$. The substitution $y = x^2$ made above, "hides" this symmetry, that is, we must remember that if we find n nonzero solutions for equation (9) there will actually be $2n$ signed nonzero solutions for x due to the hidden symmetry.

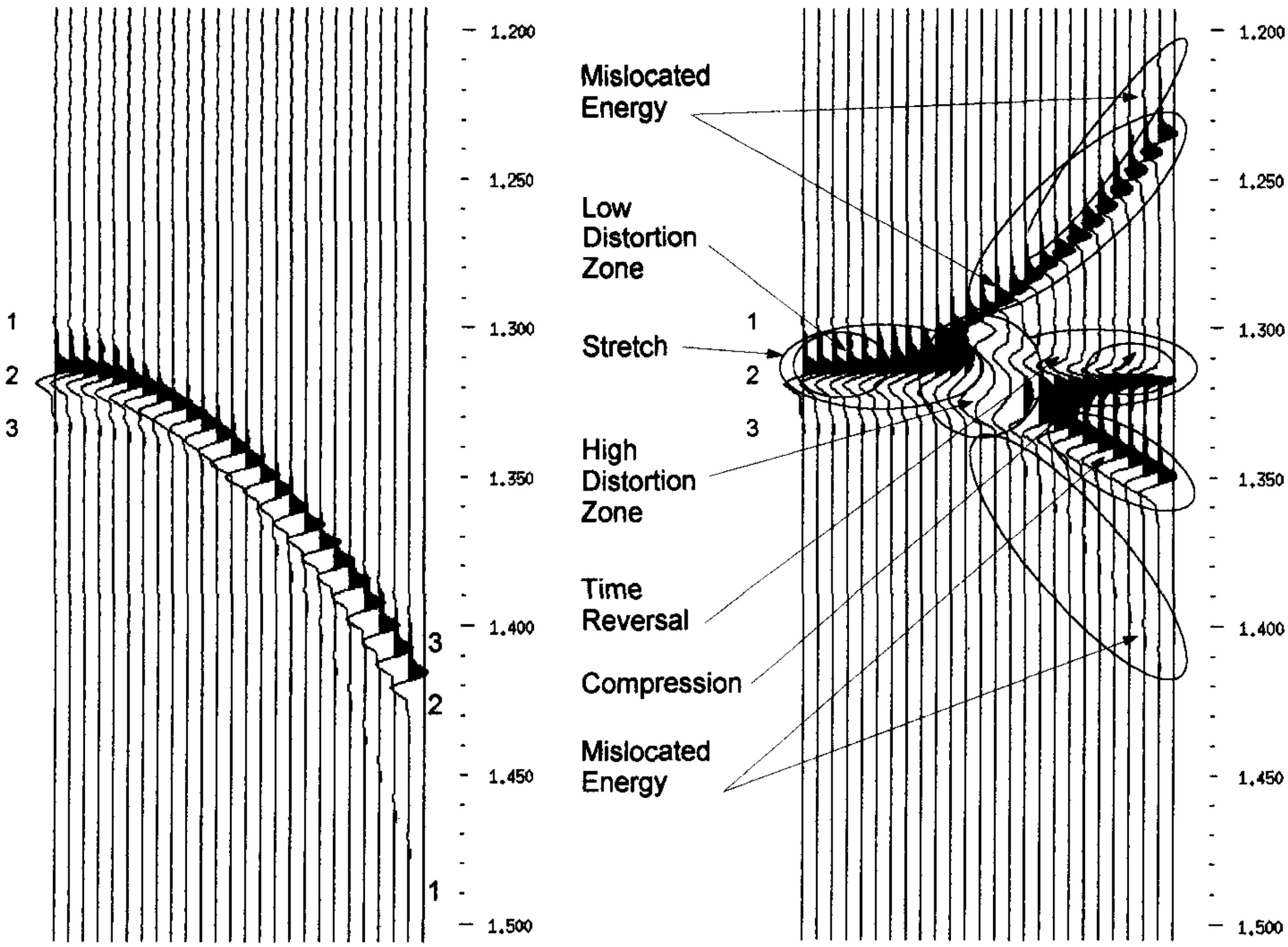

Fig. 2. Global distortion: left – model; right – model + NMO.

After some tedious elementary algebraic manipulation, equation (9) leads to a more tractable standard expression, namely:

$$ay^2 + by + c = 0\,, \tag{10}$$

where

$a = (\sigma_2 - \sigma_1)^2$;
$b = 2(\sigma_1(T_1 - k - T_2) + \sigma_2\,(T_2 + k - T_1))$; and
$c = (T_2 - T_1)^2 + 2k(T_2 - T_1) + k^2$,
with $k = -(\beta^2)$.

We are interested in real values of y such that $0 < y < \infty$.

We note here that equation (10) will in fact be valid also for the compression case as we can see by comparing equations (7) and (8). The point is that in one case for equation (9) we will have $\beta = (P + 1)\Delta t(0)$ and for the other have $\beta = -(P + 1)\Delta t(0)$ but only β^2 appears in the equation that determines the coefficients a, b and c of the quadratic.

Equation (10) is the standard quadratic equation which has the well-known algebraic solution(s):

$$y = (-b \pm (b^2 - 4ac)^{1/2}) \,/\, 2a\,, \qquad a \neq 0.$$

Once y is known, the required symmetric signed offsets x may be calculated from $x = \pm y^{1/2}$. If there are two real values of y which satisfy equation (10), then we must be careful to select the smaller one as the NMO-offset constraint [see case (c) below].

A Complete Set of Four Subclasses for NMO Distortion

Case (a): $v_1 = v_2$ – the monotonic convergence case

It is clear by back substitution that $a = 0$ if and only if $v_1 = v_2$. In this case, equation (10) reduces to the linear relationship:

$$by + c = 0, \tag{11}$$

which can have only one solution for y, namely $y = -c/b$. Since there is at most one solution and we know that in this case the two curves are both asymptotic to the same line, it is clear that for this case the two curves must converge monotonically.

Case (b): $v_1 > v_2$ – the monotonic divergence case

We know that if $v_1 > v_2$, the two NMO curves diverge for far offsets and that divergence causes a compression distortion. The condition for monotonic divergence is that equation (10) have only one real root with $v_1 > v_2$. A criteria for this case will be given in the discussion of case (c) below.

Case (c): $v_1 > v_2$ – the converge-diverge case

This is the only case in which there may be two different offsets with the same amount of distortion. The two NMO curves must first converge and then diverge since we know that for far offsets they diverge. In this case, equation (10) will have two real roots and the time difference for the two NMO curves will have a local minimum for some nonzero offset. At a local minimum, the derivative of the expression for time difference will be zero; that is, we must have

$$\partial \Delta t(0) / \partial x = 0 .$$

After substituting for $\Delta t(0)$, differentiating and rearranging we have

$$x = (\alpha)^{1/2} ,$$

where:

$$\alpha = (\sigma_2{}^2 T_1 - \sigma_1{}^2 T_2) / (\sigma_2{}^2 \sigma_1 - \sigma_1{}^2 \sigma_2) .$$

It is clear that x is real if and only if $\alpha > 0$.

After some substitution and manipulation, we observe that there can only be one consistent set of conditions for which $\alpha > 0$:

$$s_1/s_2 < t_1(0)/t_2(0) \text{ and } s_1 < s_2 .$$

These conditions are equivalent to $v_2/v_1 < t_1(0)/(t_1(0) + \Delta t)$ and $v_1 > v_2$ where we define $\Delta t = t_2(0) - t_1(0)$ so that $\Delta t > 0$. This set of conditions is very useful when we examine the local distortion for time $t_1(0)$ in which case we identify Δt with the sample interval.

If there is no local nonzero real minimum, it is clear that the two curves must diverge monotonically and the global minimum time difference is at zero offset. This is case (b) above for which we have the criteria:

$$v_2/v_1 \geq t_1(0)/(t_1(0) + \Delta t) \text{ and } v_1 > v_2 .$$

Using these criteria, it is easy to display a distortion classification chart for any NMO velocity function and a given trace sample rate. Figure 3 is an example of such a chart.

Case (d): $v_1 < v_2$ – the crossover case

In this case, we know that the curves will cross and that we must apply NMO corrections only for offsets less than the crossover distance in order to avoid catastrophic errors. It is also clear that the NMO distortion will be of the stretch type inside the crossover offset. Since we saw in case (c) above that there can be more than one real root only for cases in which $v_1 > v_2$ and $t_2(0) > t_1(0)$, we see that the two curves must monotonically converge inside the crossover distance and monotonically diverge in the time reversal zone beyond the crossover distance.

Table 2. Complete NMO distortion classification.

Case	Type	Criteria	Comment
(a)	Monotonic–Converge	$v_2 = v_1$	Stretch $x_p = \pm(-c/b)^{1/2}$
(b)	Monotonic–Diverge	$v_2 < v_1$ $v_2/v_1 < t_1(0)/(t_1(0) + \Delta t)$	Compression
(c)	Converge–Diverge	$v_2 < v_1$ $v_2/v_1 \geq t_1(0)/(t_1(0) + \Delta t)$	Compression ($x <$ minimum) Minimum at $(\alpha)^{1/2}$
(d)	Crossover	$v_2 > v_1$	Stretch ($x <$ crossover) Cross at $((T_2{}^2 - T_1{}^2)/(s_1{}^2 - s_2{}^2))^{1/2}$

Much of the information contained in Table 2 is graphically illustrated in Figure 3 in which a specific NMO velocity function is analyzed. The basic local distortion chart can be a valuable tool for analysis of distortion, particularly if other curves are presented on the same chart. Some of the more obvious possibilities are (using colour): (1) plot several velocity curves together; (2) plot distortion for different values of zero offset time differences; and (3) plot crossover distances.

SUMMARY

The standard NMO equation $t(x) = (x^2/v^2 + t^2(0))^{1/2}$ is equivalent to the equation $t^2(x) = x^2/v^2 + t^2(0)$ which we recognize as an hyperbola. Although this is a nonlinear relationship between the variables x and t, its familiarity and apparent simplicity can easily lull the user into a false sense of security. When using any nonlinear relationship, we must be constantly on guard for unexpectedly complex properties and difficult numerical behaviour.

We should keep in mind that most of classical physics may be described with nothing more complex than second-order equations and that these equations are capable of representing some very (at first glance) nonintuitive phenomena. Recent numerical and analytical research has revealed, for instance, that very exotic computational problems are encountered when computing orbits using ellipses and other conics. This is still an active area of research despite the fact that this is one of the first problems "solved" using the analytical methods developed by Newton.

When a nonlinear relationship is used as the basis for a critical processing step involved in data reduction, we should automatically suspect that the position of the process (especially relative to other nonlinear processes) in the flow should be carefully evaluated. This follows from the basic mathematical fact that nonlinear operations do not in general commute. In the case of NMO application for seismic data, since this is the single most critical step for determining the final quality of the stacked data to be interpreted, only the very safest and conservative practices known to the user and software developer should be used.

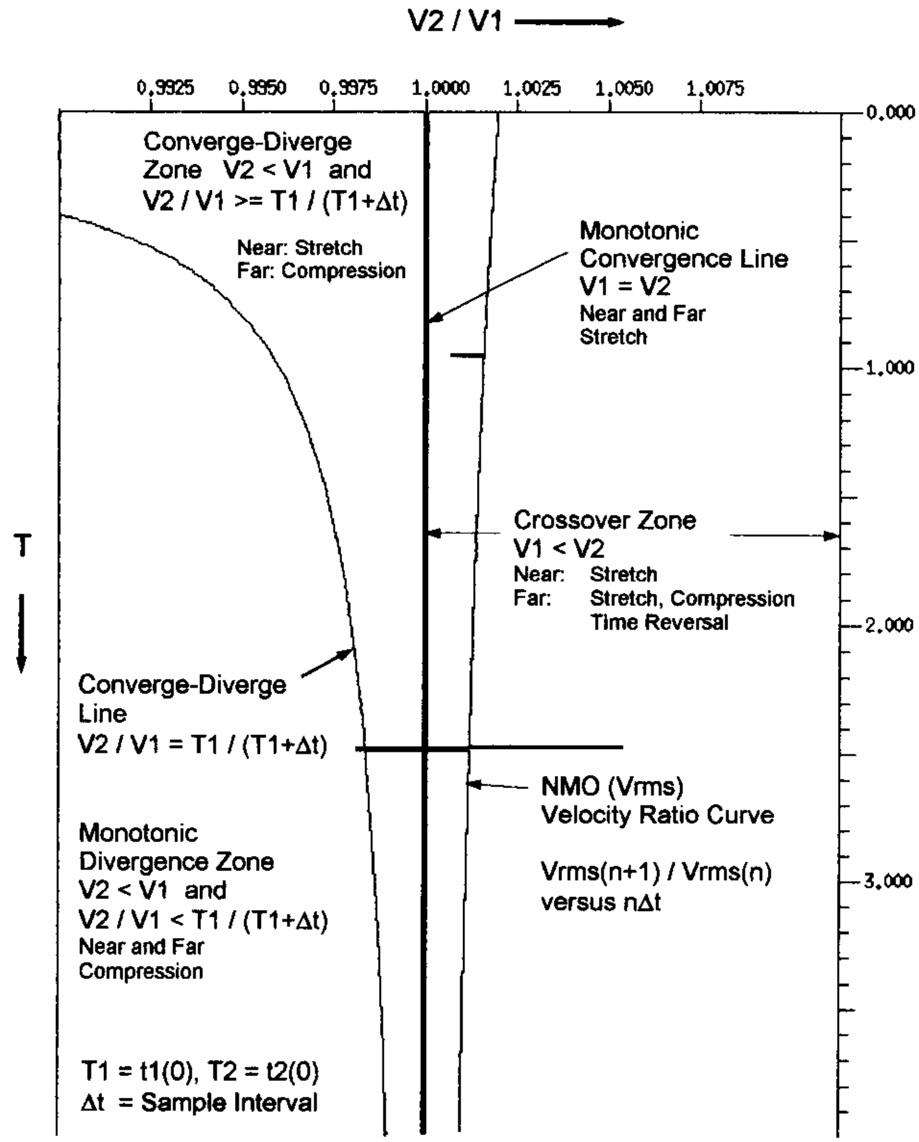

Fig. 3. Local distortion chart.