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BENEFITS OF AN OMNIDIRECTIONAL ELECTROMAGNETIC SYSTEM IN CONDUCTIVITY MAPPING

K. DUCKWORTH[1], E.S. KREBES[1], E.O. AYDEMIR[1] AND M. CHUNG[1]

ABSTRACT

The performance of an electromagnetic exploration device employing the horizontal coaxial coil configuration is examined by means of physical scale modelling and by development of relevant theory in the context of the application of this coil configuration to low induction number conductivity mapping in environmental surveys. Comparison of the performance of the horizontal coaxial coil configuration with the commonly employed horizontal coplanar and vertical coplanar configurations demonstrates that it provides an omnidirectional response which is free of the directional effects inherent in the responses provided by the other coil systems. When applied to depth sounding the horizontal coaxial coil configuration is shown to provide a depth of exploration intermediate to the depths of exploration provided by the horizontal and vertical coplanar coil configurations.

INTRODUCTION

Low induction number (LIN) electromagnetic systems, as described by Wait (1962) and McNeill (1980), are notably effective in the production of conductivity maps related to the monitoring of the environment and in archeological applications as described by Tabbagh (1986). In this type of application it is desirable that the resulting maps be unaffected by the configuration of the device employed in the survey, yet the horizontal separation of the coils in presently available LIN systems causes the responses of these systems to contain strong directional components. Thus, when conductivity maps are generated by these devices the results are dependent on the azimuth of the device that is used in the mapping survey. LIN systems normally employ the horizontal or vertical coplanar coil configurations but the purpose of the present discussion is to argue that an alternative coil configuration, namely the horizontal coaxial configuration, in which the coils are separated in the vertical rather than the horizontal direction, offers the advantage of omnidirectional response. To this end, we present appropriate theory for the use of such a configuration and physical scale model responses which demonstrate the omnidirectional character of its response by comparison with the response of the conventional coil configurations used over the same targets.

A portable version of a device employing the horizontal coaxial coil configuration would inevitably be limited to coil separations no greater than 4 m. The carrying of such a vertically oriented beam over a survey site should be no more difficult than carrying a beam of comparable dimensions in the horizontal attitude as is a feature of one commonly used LIN device. It may be noted that magnetic gradiometers employ vertical beams of comparable dimensions. The vertical beam may be notably easier to use on sites where bushes and small trees may be encountered. Where the user is interested only in the soil layer, the length of the vertical beam would be of the order of one metre.

In the context of depth sounding, the expansion of horizontally separated coils will inevitably be more practical than vertical expansion of coaxial coils, but practical merits of vertical expansion would be the ability to conduct a sounding on a site with limited lateral access and the freedom from the geologic noise that contaminates horizontal expansion soundings as the individual coils pass over localized conductivity discontinuities. Thus, as such a system may be of some practical interest, theory for depth sounding with such a device is presented later. There is no doubt that the device would not be portable but it appears possible that vertical expansion of coil separation by up to 30 m would be possible on a mast which could be relatively quickly erected at a survey site.

MODELLING RESULTS

The directional effects observed in the responses of horizontally separated coils are particularly clear in the case of localized conductivity discontinuities with dimensions comparable to the coil separation and which are located at depths of the order of one tenth of the coil separation. On maps derived from the responses of horizontally separated coils, such targets create a double anomaly in which the separation of the anomalies depends on the coil separation. This effect is caused by the tendency for the transmitter and receiver coils to respond separately to shallow localized

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[1]Department of Geology and Geophysics, The University of Calgary, 2500 University Drive N.W., Calgary, Alberta T2N 1N4

conductors as was discussed by Frischknecht et al. (1991, p. 108).

An example of this directional effect in a survey over a shallow target which might represent a discarded buried metal plate is presented in Figures 1a and 1b. These responses were obtained by means of physical scale modelling. The model target had dimensions of 12 cm by 8 cm with a 480 S conductance. The model frequency was 400 kHz with the traverse lines at 2 cm spacing and a station spacing of 2 cm. The traverse data was converted into map form by a two-dimensional sinc interpolation with the results presented as a bit map at EGA screen resolution (640 x 350 pixels). The maps could have been produced by having the modelling system acquire the responses at station and line densities sufficiently high to eliminate the need for interpolation but this would have greatly increased the modelling time. With the station and line densities used here, each map represents two hours of operation on the modelling system. Sinc interpolation is readily applicable to data acquired in field surveys provided the data are acquired with constant line and station spacing. although line and station spacings do not have to be equal as they were in this case.

Figure 1a shows a map of the inphase response of a system employing horizontal coplanar coils passing over a horizontal rectangular plate located at a depth equal to one sixth of the coil separation, while the response for the same plate at the same depth below the coils but with the coil separation increased by 50% is shown in Figure 1b. The quadrature component responses which are normally the basis of LIN surveys are not shown as the target in this case gave very little quadrature response due to its high conductance. The only purpose of these illustrations is to demonstrate the directional effect which must inevitably be present in both the inphase and quadrature data. In these cases the anomalies can be viewed as double images of the target with the separations of the images depending on the coil separations. The stacked profiles presented to the left of each map are included to demonstrate that presentation of the data in profile form does not permit the double image effect to be readily appreciated. The shading used on the profiles is an attempt to relate the magnitudes on the profiles to the contours on the maps but this can only be truly appreciated when using an active computer display of the anomalies. In each map the actual target lies midway between the images. It is evident that given sufficient coil separation, a survey of this type could produce completely separate images of the target. Thus, in the context of the intention to map the distribution of subsurface conductivity, these results could be very misleading. The map produced by the proposed omnidirectional horizontal coaxial coil configuration over this same target using the same model coils separated vertically is shown in Figure 1c. As expected, the image of the plate is free of the double image effect and in the modelling system it was located immediately over the target. Variation of the coil separation with the lower coil at a fixed height above the target only changed the magnitude of the anomaly and to some extent its width.

Even greater confusion of the response can arise when a horizontally separated coil system passes over closely spaced but electrically isolated targets as shown in Figure 2. In this case, the targets were two identical horizontal metal cylinders such as might represent discarded metal drums in a landfill. When the coil separation was equal to the gap between adjacent sides of the targets (Figure 2a), the resulting anomalies were well resolved, but increasing the separation until it equalled the separation between the centres of the targets (Figure 2c) caused the responses of the two targets to merge and to present the impression that the most conductive zone lay at the midpoint between the targets. The corresponding omnidirectional response for this target when the coils were in the horizontal coaxial vertically separated configuration is shown in Figure 2d. The response in this case displays good resolution of the anomalies, as is to be expected. Changing the coil separation produced only a change in anomaly magnitude and a slight loss of the resolution of the two anomalies.

A second type of directional effect occurs when the conductivity discontinuity is elongated. If the long axis of the target lies parallel to the azimuth of the horizontally separated coil system, the response will be markedly different than if the elongation lies parallel to the coil system azimuth. This effect is familiar in mineral exploration where elongated steeply dipping vein-type conductors are known to display this effect, as was shown by Frischknecht et al. (1991, p. 127, 128). Steeply dipping conductive structures will not commonly be met in environmental surveys but flat-lying elongated ribbons of conductor such as may be represented by a buried channel will be common. An example of the response of such a flat-lying ribbon of conductor is shown in Figure 3. The horizontal coplanar coil response for a coil azimuth perpendicular to the strike trend of the conductor is shown in Figure 3a while the parallel azimuth or broadside response is shown in Figure 3b. The broadside response correctly indicated a single conductor but indicated the conductor to be longer than the actual target. The perpendicular response indicated two apparent conductors with the actual conductor located midway between the two anomalies. It is also notable that as the ribbon was longer than the coil separation in the broadside configuration, both coils could be simultaneously located over the target, and this resulted in a clearly stronger response than that seen with the perpendicular azimuth. The horizontal coaxial response to this target shown in Figure 3c indicated a target of the correct length located directly below the anomaly.

SUMMARY OF THEORY

McNeill (1980) presented simple theoretical expressions by which the layered and homogeneous earth responses of horizontal and vertical coplanar coil systems could be calculated under low induction number conditions. Induction number B is defined as:

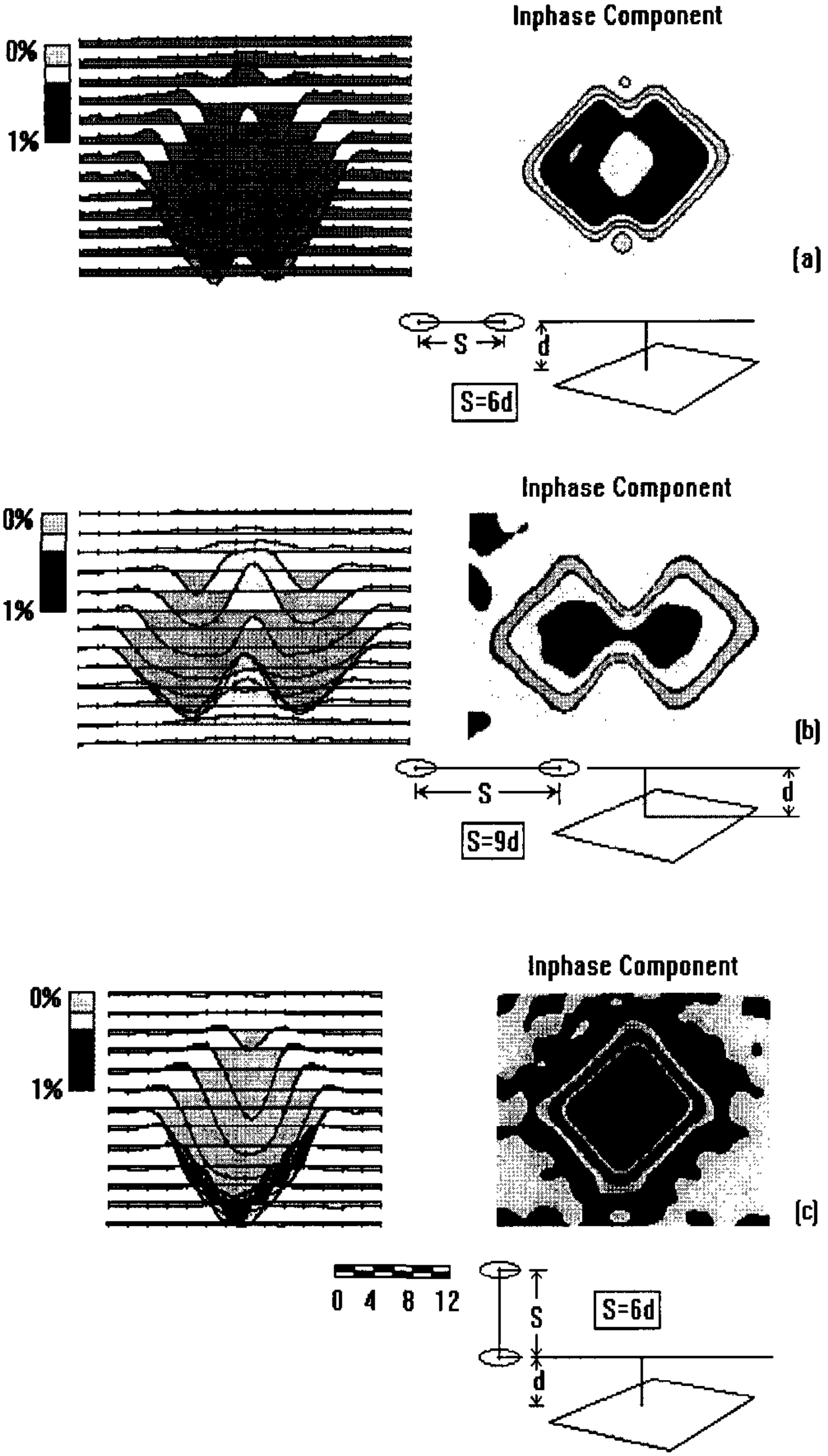

Fig. 1. Illustrations **(a)** and **(b)** are responses of a horizontal rectangular plate as observed with horizontal coplanar coils. Illustration **(c)** is the response of the same target at the same depth using the same coils in the horizontal coaxial configuration.

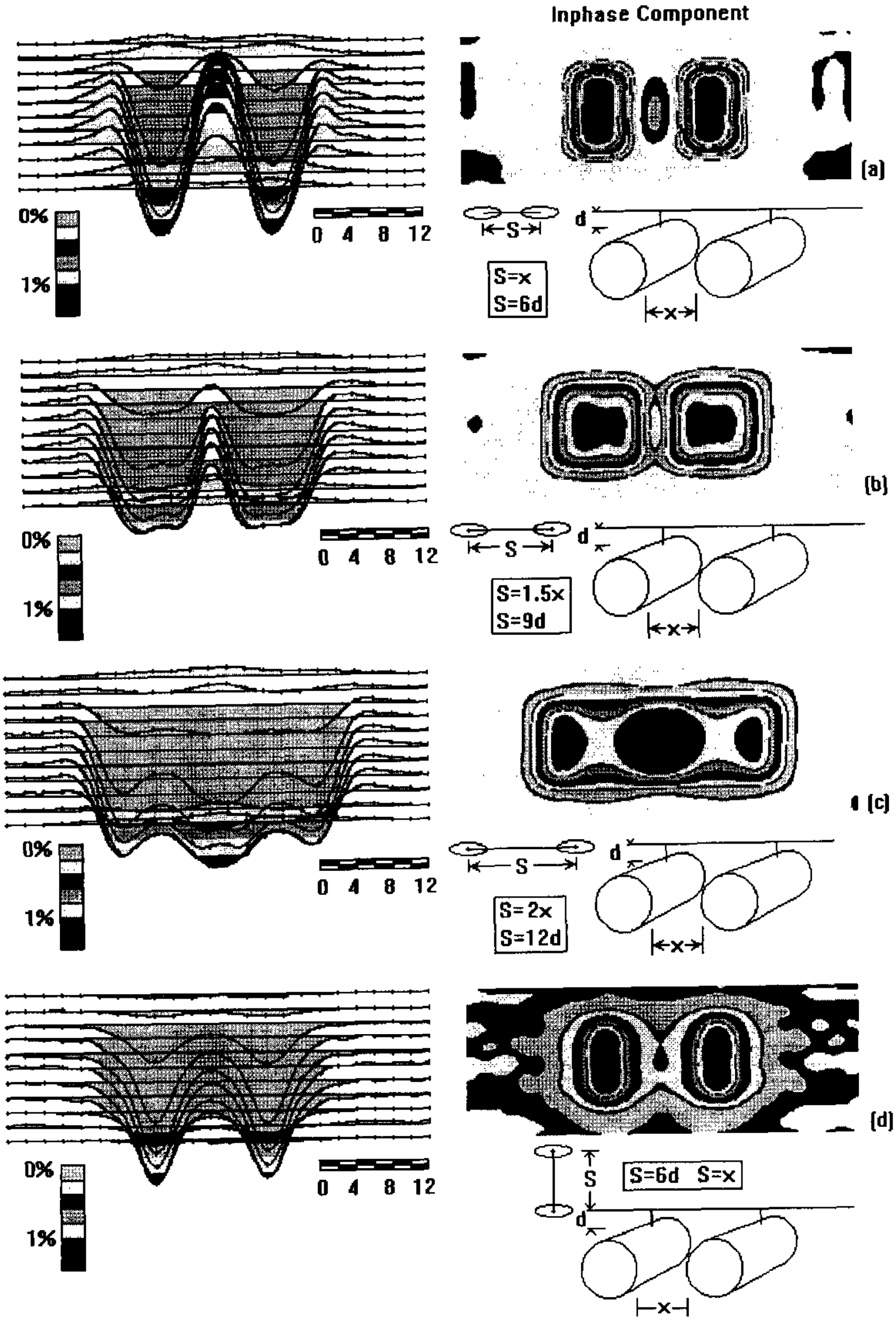

Fig. 2. The responses of closely spaced but electrically isolated conductors **(a)**, **(b)** and **(c)** depend strongly on the relationship of the separation of the horizontal coplanar coils with respect to the separation of the targets. The responses provided by the same coils in the horizontal coaxial configuration **(d)** provide a notably clearer response.

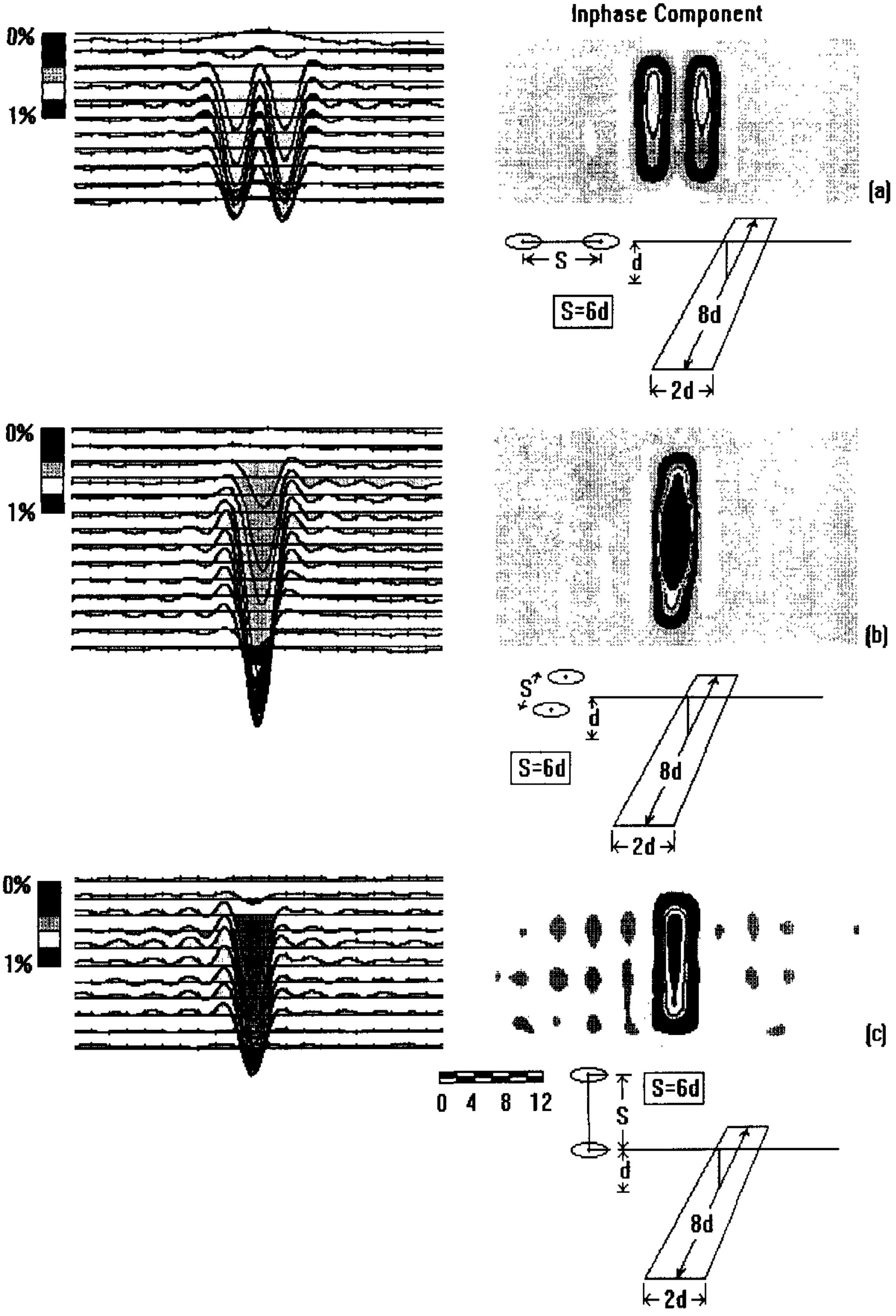

Fig. 3. A flat-lying ribbon-type conductor appears as two conductors **(a)** if horizontal coils pass over the conductor perpendicular to strike. If the coils cross the target broadside **(b)** the target appears as a single conductor in the map but it is elongated. The horizontal coaxial response provides a clear indication of a single conductor and provides a good indication of the length and true position of the conductor.

$$B = \frac{\text{Coil Separation}}{\text{Skin Depth}} = S\sqrt{\frac{\sigma\mu_0\omega}{2}} = S/\delta, \qquad (1)$$

where S is the separation between the coils, σ is conductivity, μ_0 is the magnetic permeability of free space, ω is angular frequency and δ is skin depth. Low induction number conditions imply that coil separation is of the order of or less than one tenth of the skin depth.

In the homogeneous earth case McNeill (1980) demonstrated that for LIN conditions the response of the system is linearly related to conductivity and that the expression relating the conductivity to the response of the system is the same for the horizontal and vertical coplanar coils configurations, this being:

$$\sigma_a = \frac{4}{\omega\mu_0 S^2}\left(\frac{H_s}{H_p}\right)_{\text{HorizontalorVerticalCoplanarQuadrature}}, \qquad (2)$$

where (H_s/H_p) is the quadrature response of the system. For a realistic earth this expression produces an apparent conductivity so that conductivity has been designated as σ_a rather than σ.

In the case of the horizontal coaxial system, theory detailed later demonstrates that the following expression applies:

$$\sigma = \frac{8}{\omega\mu_0 S^2}\left(\frac{H_s}{H_p}\right)_{\text{HorizontalCoaxialQuadrature}}, \qquad (3)$$

where, again, (H_s/H_p) is the quadrature response of the system.

In the layered earth case, McNeill showed that the apparent resistivity for a three-layer earth can be calculated by means of the following expression:

$$\sigma_a = \sigma_1\left[1 - R_{V/H}(z_1)\right] + \sigma_2\left[R_{V/H}(z_1) - R_{V/H}(z_2)\right] + \sigma_3\left\{R_{V/H}(z_2)\right\}, \qquad (4)$$

where the function $R_{V/H}(z_n)$ was described by McNeill as the cumulative response function.

In the case of horizontal coplanar coils, McNeill provided the following expressions for the cumulative response function:

$$R_v(z_n) = \frac{1}{\left(4z_n^2 + 1\right)^{1/2}} \qquad (5)$$

and for vertical coplanar coils that:

$$R_H(z_n) = \left(4z_n^2 + 1\right)^{1/2} - 2z_n. \qquad (6)$$

In the case of horizontal coaxial coils we demonstrate later that the expression for R becomes:

$$R_V(z_n) = \frac{1}{2z_n + 1}, \qquad (7)$$

where, in all cases, z_n is the ratio of layer depth to coil separation.

A graphical display of these three functions for z_n ranging from 0 to 3 times the coil separation is provided in Figure 4. McNeill noted that this type of comparison indicated that the horizontal coplanar configuration offers approximately twice the effective exploration depth of the vertical coplanar configuration. The plot of the response for the horizontal coaxial

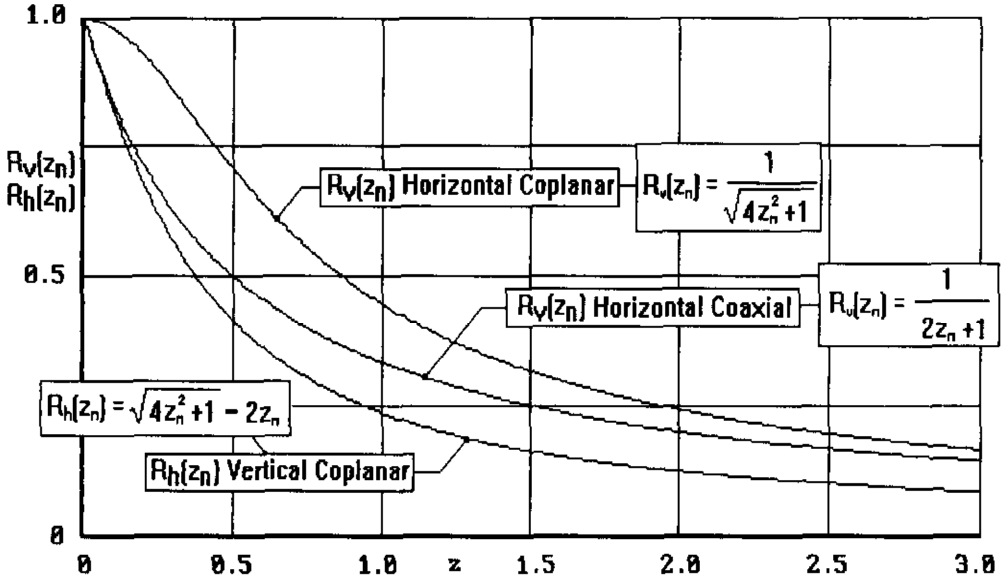

Fig. 4. The cumulative response functions for three coil configurations raised above a homogeneous half-space indicate that the horizontal coaxial configuration will provide a depth of exploration intermediate between those of the horizontal and vertical coplanar configurations.

coils lies intermediate to the responses provided by the horizontal and vertical coplanar configurations so that we may infer that the horizontal coaxial configuration has an effective depth of exploration superior to that of the vertical coplanar configuration. The display indicates that for larger values of z_n the horizontal coaxial configuration has an effective depth of exploration which approaches that of the horizontal coplanar configuration.

A comparison of the depth sounding responses that these three coil configurations produce over the same three-layer earth are presented in Figure 5. In this case, the resistivity of layer 2 was allowed to vary over a two-decade range from 10 to 1000 ohm-m while the resistivities of layers 1 and 3 were fixed at 100 ohm-m with the thickness and depth of layer 2 remaining fixed. These results indicate that the horizontal coaxial configuration will respond to a layered structure in a manner similar to that displayed by the vertical coplanar configuration but that the response to layer 2 will be somewhat stronger for the horizontal coaxial coils at smaller coil separations, particularly when layer 2 is a better conductor than layer 1. The horizontal coplanar response to this structure is notably stronger than the responses of either of the other two configurations. It is unlikely that vertical expansion of the coil separation will ever be employed beyond 30 m but Figure 5 suggests that useful layering information could be obtained within this limitation.

THEORY FOR HORIZONTAL COAXIAL COILS RAISED ABOVE A HOMOGENEOUS HALF-SPACE

Following Wait (1982, p. 113) the vertical component of the primary and secondary magnetic fields in the region above a layered half-space due to a horizontal source coil located as illustrated in Figure 6 are given by:

$$\left(H_z^p\right)_{r=0} = \frac{2C}{S^3} \tag{8}$$

and

$$\left(H_z^s\right)_{r=0} = -\frac{C}{\delta^3}\int_0^\infty \bar{R}(g)g^2 e^{-gA}dg = -\frac{C}{\delta^3}\left[T_0\right]_{r=0}, \tag{9}$$

where $C = I/(4\pi)$, the decay constant $A = (z + h)/\delta$ and skin depth $\delta = \sqrt{2/\sigma\mu_0\omega}$.

The integral $[T_0]$ was shown by Wait (equation 146) to be given by:

$$\left[T_0\right]_r = \int_0^\infty\left[\frac{\sqrt{g^2+2i}-g}{\sqrt{g^2+2i}+g}\right]g^2 e^{-gA}J_0(gB)dg, \text{ where } B = r/\delta. \tag{10}$$

In the case of the horizontal coaxial coil configuration where the observation point is located on the axis of the source coil the Bessel function takes the value 1 so that we have

$$\left[T_0\right]_{r=0} = \int_0^\infty\left[\frac{\sqrt{g^2+2i}-g}{\sqrt{g^2+2i}+g}\right]g^2 e^{-gA}dg = \int_0^\infty I_e(g)dg. \tag{11}$$

This integral can not be evaluated in any reasonably simple closed form but for large values of g the integrand is closely approximated by the function

$$I_a(g) = \frac{i}{2}e^{-gA}. \tag{12}$$

Computation of the exact integrand I_e and of the function I_a for a range of values of g demonstrates that these functions only diverge significantly when g lies in the range 0 to 3. In low induction number conditions the decay constant A will be small so that both the exact and approximate functions will decay very slowly with g. This implies that the integral of the approximate function I_a from 0 to ∞ will be almost identical to the exact integral. Thus, we may use the following approximation when A is small:

$$\frac{\left(H_z^s\right)_{r=0}}{\left(H_z^p\right)_{r=0}} = -\frac{S^3}{2\delta^3}\left[T_0\right]_{r=0} = -\frac{1}{2}B^3\left[T_0\right]_{r=0} \approx -\frac{1}{2}B^3\left[\int_0^\infty \frac{i}{2}e^{-gA}dg\right] = -\frac{B^3}{2}\left[\frac{i}{2A}\right], \tag{13}$$

where B is redefined as $B = |z-h|/\delta = S/\delta$.

Substituting $A = (z+h)/\delta = (2h \pm S)/\delta = \left[2(h/S)\pm 1\right]B$ we have

$$\frac{\left(H_z^s\right)_{r=0}}{\left(H_z^p\right)_{r=0}} = -\frac{i}{4}\frac{B^2}{\left[2(h/S)\pm 1\right]}. \tag{14}$$

In this analysis, the distance h is the distance of the source coil above the half-space and the ±1 relates to the cases of the receiver being (+) above or (–) below the transmitter. If we replace h with h', which is the height of the lower coil regardless of which coil is lower, then the negative case can be discarded so that

$$\frac{\left(H_z^s\right)_{r=0}}{\left(H_z^p\right)_{r=0}} = -\frac{i}{4}\frac{B^2}{\left[2(h'/S)+1\right]} = -\frac{i}{4}\frac{B^2}{\left[2z_n+1\right]}. \tag{15}$$

If the source coil is located on the surface of the half-space $h' = 0$ and the response becomes

$$\frac{\left(H_z^s\right)_{r=0}}{\left(H_z^p\right)_{r=0}} = -\frac{i}{4}B^2 = -\frac{i}{4}\frac{S^2}{\delta^2} = -\frac{i}{4}S^2\frac{\sigma\mu_0\omega}{2} = -\frac{iS^2\sigma\mu_0\omega}{8}. \tag{16}$$

Thus, the conductivity of a homogeneous half-space or the apparent conductivity of a layered half-space may be derived from the quadrature response of horizontal coaxial coils by means of the expression

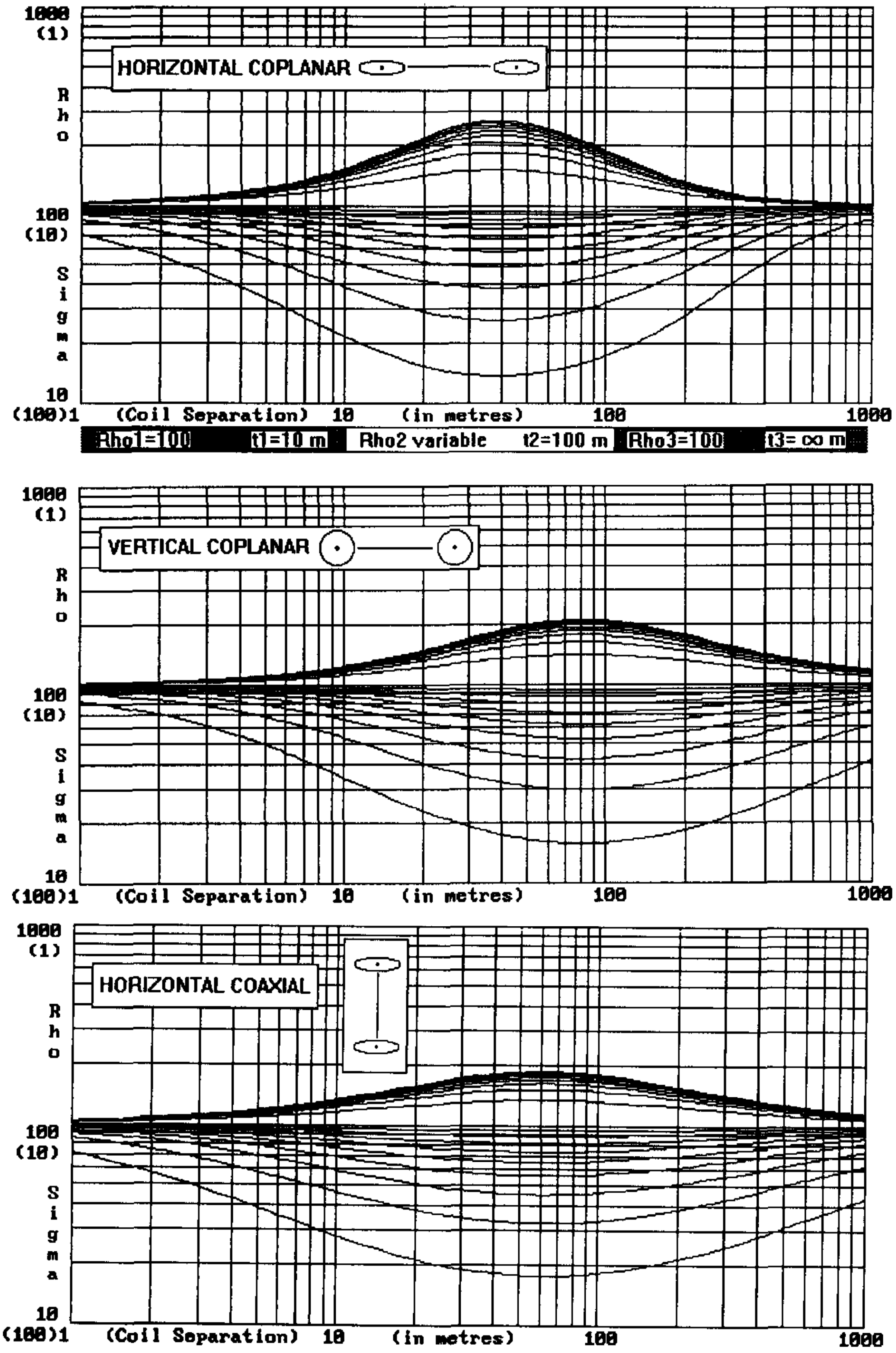

Fig. 5. Depth sounding profiles computed over a three-layer earth for the three coil configurations where the second layer resistivity was varied between 10 and 1000 ohm-m indicate that the horizontal coaxial system will perform in a manner very similar to that of the vertical coplanar configuration.

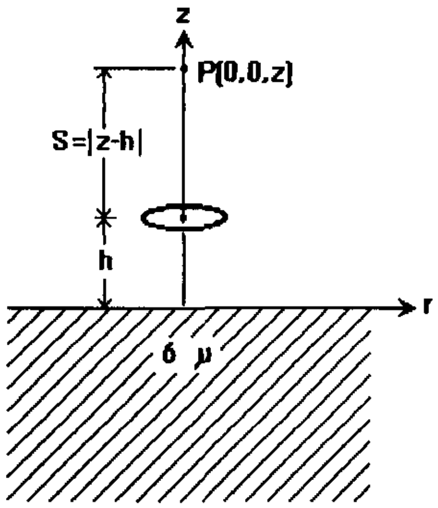

Fig. 6. Geometry of a horizontal source coil raised above a homogeneous half-space.

$$\sigma_{\text{apparent}} = \frac{8}{\omega\mu_0 S^2}\left(\frac{H_z^s}{H_z^p}\right). \tag{17}$$

McNeill (1980) defines the function $R_v(z_n)$ as the relative contribution to the secondary magnetic field from all material below a depth z. Thus,

$$R_v(z_n)=\left(\frac{H_z^s}{H_z^p}\right)_z / \left(\frac{H_z^s}{H_z^p}\right)_0 = \left(-\frac{i}{4}\frac{B^2}{\left[2z_n+1\right]}\right) / \left(-\frac{i}{4}B^2\right) = \frac{1}{\left[2z_n+1\right]}. \tag{18}$$

CONCLUSIONS

The horizontal coaxial coil configuration appears to have immediate potential as a portable device aimed at conductivity mapping. Its outstanding benefit will be the omnidirectional response that it will provide. When applied to detailed mapping within the top metre the device should be notably convenient in that if the coils are sufficiently compact it could take the dimensions and mode of operation of a walking stick. A less likely application would be as a depth sounding system, although this would be possible and probably less prone to geologic noise than the conventional systems in addition to offering the opportunity to conduct soundings on sites which might otherwise be unsuited to sounding. However, the practical difficulties of expanding coils vertically will be considerable. Theory presented here permits the immediate application of the device in the mapping and depth sounding modes and it is hoped that this discussion will encourage the use of this configuration in environmental monitoring.

Omnidirectional devices already exist in TEM systems so that the considerations presented here could be applied with such systems, but the large loop transmitters that are characteristic of TEM systems are not well suited to the detailed type of near-surface mapping that is necessary in environmental surveys.

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