CANADIAN JOURNAL OF EXPLORATION GEOPHYSICS
VOL. 31, NOS 1 & 2 (DECEMBER 1995), P. 11-17

P-WAVE VELOCITY AND DENSITY ESTIMATES FROM THE LINEAR INVERSION OF VSP DATA

ALEXANDER S. TSEMAHMAN[1], MICHAEL J. JONES[2] AND FRANTISEK HRON[1]

ABSTRACT

The determination of elastic parameters near the well from vertical seismic profile (VSP) data is important if sonic or density logs are not available, or for an independent confirmation of the estimated elastic parameters obtained from logs. Most of the existing standard methods of linear inversion use traveltimes only, thus providing estimates of the seismic velocity. Alternatively, the waveforms may be inverted, the result usually being the impedance distribution. However, the latter approach is very computer-intensive even for a one-dimensional problem. We used a ray-based inversion, thus preserving the high speed of the conventional method, but also included the amplitude information in the inversion scheme. The solution of the seismic forward problem was found using asymptotic ray theory. We linearized the nonlinear forward problem and applied a linear iterative procedure based on singular value decomposition to find estimates of *P*-wave velocities from transit times of downgoing *P*-waves. We also used the amplitudes of the vertical components of these waves to invert for density. The results of traveltime and amplitude inversion of field VSPs were compared with well logs. Inversion gave reasonable velocity and density profiles even when the number of observations in the upper part of the well was less than the number of unknown elastic parameters preventing the use of a layer stripping method. The resolution of the amplitude inversion may be improved by using more geophones above the target zone.

Introduction

The determination of elastic parameters near the well by inversion of vertical seismic profile (VSP) data provides an independent verification of estimates of these parameters obtained from log data. Inversion is especially useful if some logs or parts of them are not available. Several authors have recently applied various inversion techniques to the transit times or full waveforms, mainly to evaluate the seismic velocity variation with depth. Stewart (1984) used the Levenberg-Marquardt method to determine *P*- and *S*-wave velocity profiles from the transit time VSP data with offsets varying from 30 to 300 m in different surveys. Lines et al. (1984) applied a linear iterative inversion based on layer stripping and singular value decomposition (SVD) to estimate layer dips near the well by using the transit times of direct *P*-wave arrivals from a 305 m offset VSP survey and a sonic log as a horizontally layered starting model. Pujol et al. (1985) determined the *P*-wave velocity profile from a 181.3 m offset VSP using the Levenberg-Marquardt method and explicit formulae for the Fréchet matrix. Grivelet (1985) computed the impedance variation with depth from inversion of upgoing *P*-wave, but his method is suitable only for the case of normal incidence in a horizontally layered medium. A similar method was used by Macé and Lailly (1986) who showed that the impedance for such a medium could be obtained without the separation of downgoing and upgoing wave fields.

Nonlinear methods have also been applied to the VSP data. These methods do not require a starting model close to the actual distribution of elastic parameters, but they are very computer-intensive. Nolte and Frazer (1994) described a nonlinear genetic algorithm to recover slowness and impedance profiles from synthetic VSP data.

Cheng at al. (1992) showed that the linear inversion of noise-free synthetic *P*-wave amplitudes could provide information about the density rather than impedance. We describe a modification of this method and present its application to field VSP data. The solution of the inverse problem requires many forward numerical simulations. Therefore, the solution of the forward problem was based on asymptotic ray theory (Hron and Kanasewich, 1971) which provided an accurate yet fast way of finding arrival times and amplitudes of seismic signals. The computed transit times and amplitudes of unconverted downgoing *P*-waves were compared with their observed values. In the case of near-offset data the residuals were minimized using a linear iterative algorithm based on SVD (Press et al., 1992). In the case of far-offset data the Levenberg-Marquardt method was used (Marquardt, 1963; Koch, 1992).

In this paper we deal only with a vertically inhomogeneous medium, but the method can be generalized and used

Manuscript received by the Editor March 20, 1995; revised manuscript received July 27, 1995.
[1]Department of Physics, University of Alberta, Edmonton, Alberta T6G 2J1
[2]Schlumberger of Canada, 2400, 801 6th Avenue S.W., Calgary, Alberta T2P 3W2
We thank PanCanadian Petroleum Ltd. for allowing the use of the data set. The first author was employed by Schlumberger of Canada as a summer student for the duration of the project. He is grateful to Allan Campbell, Perry Katopodis, Stephane Nechtschein and Richard Pearcy for their support of this project. We also wish to thank Jeremy Gallop and our reviewers for critically reading and improving the manuscript.

for determining the elastic parameters of two-dimensional structures from VSP data. An implementation of the ray-based, linear iterative inversion of crustal refraction and wide-angle reflection data is shown in Shahriar (1986) and in Zelt and Smith (1992).

We used computed synthetic data to demonstrate different aspects of the algorithm. A field VSP data set was also inverted. We used the transit times of downgoing P-waves to find the P-wave velocity variation with depth from a near-zero offset VSP. Assuming a horizontally layered model, we also determined the P-wave velocity from the far-offset data. Both profiles were similar to the corresponding sonic log. The chi-square statistic was applied to examine the validity of the solution. The fit of the near-offset data to the solution was found to be satisfactory. The fit of the offset data could be improved by using a two-dimensional generalization of the inversion algorithm. Finally, we obtained the density estimates from the amplitudes of downgoing P-waves. The density was in the same range as the log data.

Method

We assume that the medium under consideration satisfies the following constraints:

– it is isotropic and linearly elastic;

– it consists of homogeneous layers with horizontal interfaces;

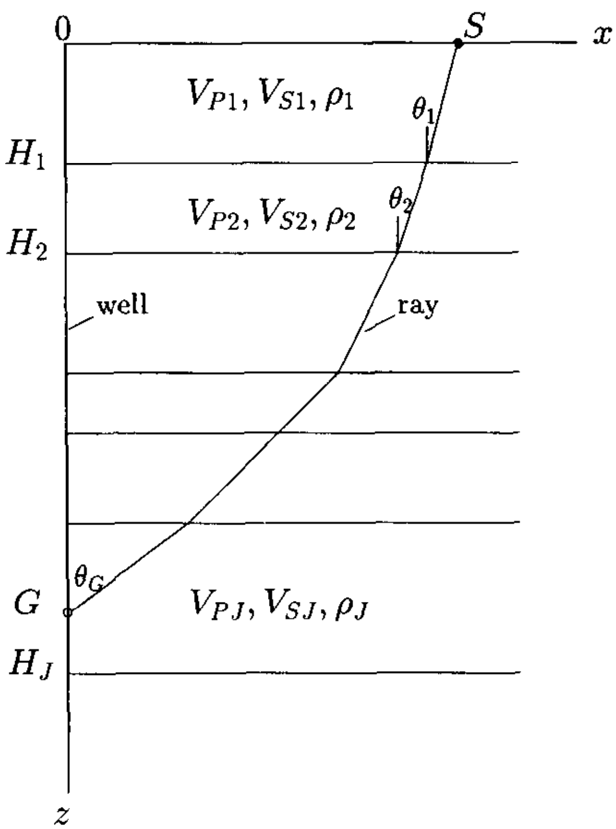

Fig. 1. Model of the medium. P-wave velocity V_P, S-wave velocity V_S and density ρ are constant within each layer.

– the distance between the source and the receiver is greater than the predominant wavelength of the signal; and

– the angles of incidence are smaller than the critical angle.

The last assumption is not crucial. It was made only for simplifying the computation of the Fréchet derivatives of the amplitudes.

The seismic sources are located on the surface and the geophones are placed in the well. Some layers (usually in the upper part of the well) may contain no geophones. The positions of interfaces are assumed to be known from the log data. Figure 1 shows a typical setup of the VSP experiment.

The forward modelling

The transit time of an unconverted downgoing P-wave recorded at the geophone is given by

$$t = \sum_{j=1}^{J} \frac{h_j}{V_{Pj}\cos\theta_j}, \tag{1}$$

assuming that the P-wave velocity V_{Pj} of the jth layer is constant. Here $h_1 = H_1$, $h_j = H_j - H_{j-1}$, $j = 2, 3, \ldots, J-1$, $h_J = H_G - H_{J-1}$. The total number of layers is denoted by J, H_j is the depth of the jth interface, h_j is the thickness of the jth layer, θ_j is the angle between the ray and the z-axis at the jth interface, and H_G is the depth of the geophone. Equation (1) can be rewritten as

$$t = \sum_{j=1}^{J} \frac{h_j}{V_{Pj}\sqrt{1-p^2V_{Pj}^2}}, \tag{2}$$

where $p = \sin\theta_j/V_{Pj}$ is the ray parameter. We differentiate t with respect to V_{Pj} to get

$$t'_{V_{Pj}} = \frac{\partial t}{\partial V_{Pj}} + \frac{\partial t}{\partial p}p'_{V_{Pj}} = -\frac{h_j}{V_{Pj}\sqrt{1-p^2V_{Pj}^2}}. \tag{3}$$

Transit times (2) and derivatives (3) are used together with the observations as the input values for the traveltime iterative inversion.

The zero-order approximation of asymptotic ray theory gives the following expression for the vertical component of particle motion in a stack of isotropic elastic layers with horizontal interfaces (Hron and Kanasewich, 1971):

$$u(s) = \frac{u(s_0)}{L}\left(\frac{z_S}{z_G}\right)^{1/2}\left(\prod_{j=1}^{J-1}\left(\frac{z_j^-}{z_j^+}\right)^{1/2} R_j\right)f(t-t_G). \tag{4}$$

Here $u(s_0)$ is the amplitude of the vertical component at a reference point on the ray at the distance s_0 from the source, z_S and z_G are the impedances at the source S and geophone G, z_j^+ is the impedance for the incident wave at the jth point of incidence, and z_j^- is the impedance at the same point, but related to the transmitted wave. We denote by $f(t)$ the time dependence of the source, by t_G the arrival time of the seismic signal at the geophone and by R_j the transmission coefficient at the jth point of incidence. The geometrical spreading is given by

$$L = \left(X \left| \frac{\partial X}{\partial \theta_S} \right| \frac{\cos\theta_G}{\sin\theta_S} \right)^{1/2} \prod_{j=1}^{J-1} \left(\frac{\cos\theta_j^+}{\cos\theta_j^-} \right)^{1/2}, \tag{5}$$

where X is the offset, θ_j^+ and θ_j^- are the angle of incidence and the angle of transmission at the jth point of incidence, whereas θ_S and θ_G denote the angles between the ray and the z-axis at the source and geophone, respectively.

Assuming that the elastic parameters (P-wave velocity, S-wave velocity and density) are constant within each layer and applying expressions (4) and (5) for computing the amplitude w of the vertical component of a downgoing P-wave, one finds for a stack of homogeneous layers:

$$w = \frac{w_0 \cos\theta_G}{L} \prod_{j=1}^{J-1} R_j. \tag{6}$$

Here $w_0 = u(t_0)\, f(t - t_0)$ is the (constant) source amplitude, and the geometrical spreading is of the form

$$L = \frac{\cos\theta_S}{V_{P1}} \left(\sum_{j=1}^{J} \frac{h_j V_{Pj}}{\cos\theta_j} \sum_{j=1}^{J} \frac{h_j V_{Pj}}{\cos^3\theta_j} \right)^{1/2}. \tag{7}$$

Standard expressions for transmission coefficients can be found in numerous texts such as Aki and Richards (1980). Using equations (6) and (7) we computed the amplitudes and found their partial derivatives with respect to the elastic parameters numerically. We then used the computed values together with the observations as input for the amplitude inversion. The inversion algorithm is described in the next subsection.

The inverse problem

A convenient method for solving the inverse problem requires the linearization of the forward problem. A review of various matrix inverse techniques can be found, for example, in Lines and Treitel (1984). We present a brief outline of the method. A Taylor series expansion of the data vector $\mathbf{d}$ in the vicinity of the initial response $\mathbf{d}_0 = \mathbf{d}(\mathbf{m}_0)$ yields

$$\mathbf{d}(\mathbf{m}) = \mathbf{d}_0 + \sum_{k=1}^{K} \left(\frac{\partial \mathbf{d}}{\partial m_k} \right)_{\mathbf{m}_0} \left(m_k - m_k^{(0)} \right), \tag{8}$$

assuming that high-order terms are small. Here $\mathbf{m}$ is the model vector and $\mathbf{m}_0$ represents the vector of initial model parameters (i.e., the starting model). Equation (8) can be rewritten in matrix form:

$$\Delta\mathbf{d} = \mathbf{D}\Delta\mathbf{m}, \tag{9}$$

where $\Delta\mathbf{m} = \mathbf{m} - \mathbf{m}_0$ is the correction vector, $\Delta\mathbf{d} = \mathbf{d}(\mathbf{m}) - \mathbf{d}_0$ is the residual vector, and

$$\mathbf{D} = \left[\left(\frac{\partial d_n}{\partial m_k} \right)_{\mathbf{m}_0} \right] \tag{10}$$

is the Fréchet matrix. It is an N by K real matrix, N being the number of observations and $K < N$ the number of model parameters.

From equation (9)

$$\mathbf{m} = \mathbf{m}_0 + \mathbf{D}^{-1}\Delta\mathbf{d}, \tag{11}$$

implying that an improved model can be found in terms of the initial model, the Fréchet inverse and the residual vector.

The singular value decomposition of an N by K real matrix $\mathbf{D}$ is any factorization of the form

$$\mathbf{D} = \mathbf{U}\Lambda\mathbf{V}^T, \tag{12}$$

where $\mathbf{U}$ is an N by K orthogonal matrix, Λ is a K by K diagonal nonnegative matrix (the matrix of singular values λ_j), and $\mathbf{V}$ is a K by K orthogonal matrix (Lanczos, 1961). The superscript ^{T} denotes transpose. Once factorization (12) is found, the generalized inverse of a rectangular matrix $\mathbf{D}$ is written

$$\mathbf{D}^{-1} = \mathbf{V}\Lambda^{-1}\mathbf{U}^T. \tag{13}$$

Substituting this into equation (11), we get

$$\mathbf{m} = \mathbf{m}_0 + \mathbf{V}\Lambda^{-1}\mathbf{U}^T\Delta\mathbf{d}, \tag{14}$$

which together with (10) provides a practical way for finding an unknown model. Since the true dependence $\mathbf{d} = \mathbf{d}(\mathbf{m})$ is nonlinear, a reasonable model $\mathbf{m}$ can be found through iterations if a starting model is close enough to the actual values of the model parameters.

The chi-square statistic was used to estimate the fit of the data to the model for the case of traveltime inversion. We assumed that the transit times t_n had a measurement error that was random and normally distributed around the time corresponding to the actual model. Let σ be the standard deviation of this Gaussian distribution. Inversion minimizes the quantity

$$\chi^2 = \frac{1}{\sigma^2} \sum_{n=1}^{N} (\Delta t_n)^2 = \frac{1}{\sigma^2} \sum_{n=1}^{N} \left(t_n - t_n(\mathbf{m}) \right)^2 \tag{15}$$

which corresponds to the maximum likelihood estimate of the model parameters $m_1, \ldots, m_K$. The ratio $\chi_\nu^2 = \chi^2/\nu$, where $\nu = N - K$ is the number of degrees of freedom and $N > K$ is frequently used as a measure of the goodness-of-fit of the data to the model (Bevington, 1969). The model is reasonable if χ_ν^2 is reasonably close to 1.

The SVD algorithm is especially useful when some singular values λ_j are zero or very small, i.e., the matrix $\mathbf{D}$ is ill-conditioned, and when other algorithms, such as Gauss-Jordan elimination, fail to find solution (13). In this case, instead of equation (14), which is equivalent to the so-called normal equation

$$\Delta \mathbf{m} = \left(\mathbf{D}^T\mathbf{D}\right)^{-1}\mathbf{D}^T\Delta \mathbf{d}, \tag{16}$$

we use the Levenberg-Marquardt method (Marquardt, 1963):

$$\Delta \mathbf{m} = \left(\mathbf{D}^T\mathbf{D}+\beta\mathbf{I}\right)^{-1}\mathbf{D}^T\Delta \mathbf{d}, \tag{17}$$

where β is a damping factor and $\mathbf{I}$ is the unit matrix. Substituting expressions (12) and (13) into equation (17) we get

$$\mathbf{m} = \mathbf{m}_0 + \mathbf{V}\left(\Lambda^2+\beta\mathbf{I}\right)^{-1}\Lambda\mathbf{U}^T\Delta \mathbf{d}. \tag{18}$$

The parameter β in equation (18) is chosen so that χ^2 is minimized. The flow chart for the traveltime and amplitude inversion is shown in Figure 2.

Application to Synthetic Data

To study the feasibility of the method we generated two noise-free, synthetic VSP data sets and applied the linear inversion algorithm to get estimates of the *P*-wave velocity and the density. The parameter β was equal to 0 in both examples. One of the synthetic examples represented a deviated well of 2200 m depth, with a surface source located at the offset of 200 m. The elastic medium below the surface consisted of 12 horizontal homogeneous layers of unknown *P*-wave velocities. The geophone positions were located from 515 to 2000 m with a spacing of 15 m. The total number of geophones was 100. We inverted simultaneously for all 12 layers. The result of traveltime inversion is shown in Figure 3. The obtained *P*-wave velocity profile coincided with the correct model. It did not depend on the initial guess if the latter was chosen in the range from about 1500 to 5000 m/s. In particular, both initial *P*-wave velocity profiles shown in Figure 3 converged to the correct model in 6 iterations.

The other synthetic example was based on a field set of VSP data. The medium was subdivided into a stack of 25 horizontal homogeneous layers. The maximum depth of the vertical well was 3375 m. The source was located 6.8 m below the wellhead at the offset of 80 m. The downhole sensors were placed from 100 to 3370 m. The total number of traces was 76. The vertical distance between two adjacent traces was about 150 m in the upper part of the well from 100 to 2200 m. Below 2200 m the increment was approximately 20 m. In this synthetic example we inverted only for density, assuming that the positions of interfaces and both *P*- and *S*-wave velocities were known and fixed. For the field example described in the next section, one indeed could keep the *P*-wave velocity fixed for each layer because the accuracy of its determination from the traveltime inversion was high. The *S*-wave velocity was not known so accurately. However, the sensitivity of the near-offset amplitude inversion with respect to the *S*-wave velocity was found to be small, with the singular values for the *S*-wave velocity

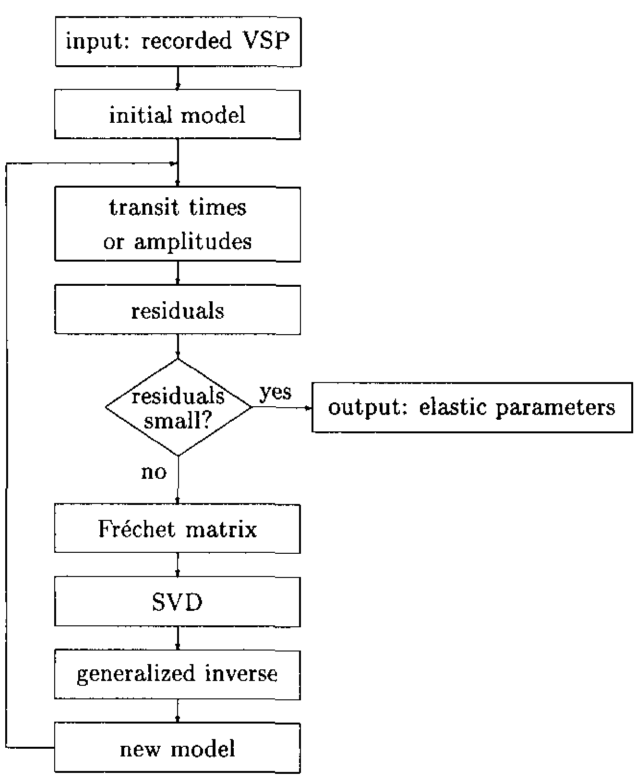

Fig. 2. Flow chart for the inversion algorithm.

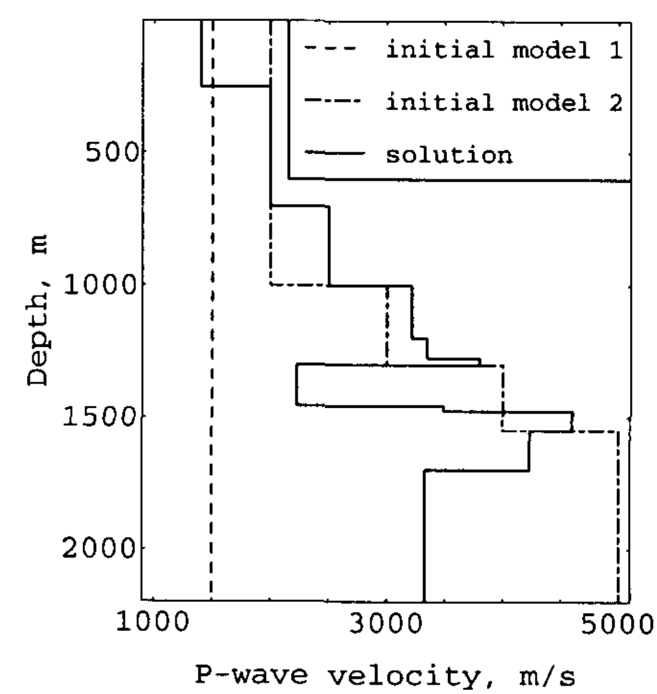

Fig. 3. Traveltime inversion of the synthetic data for *P*-wave velocity after 6 iterations.

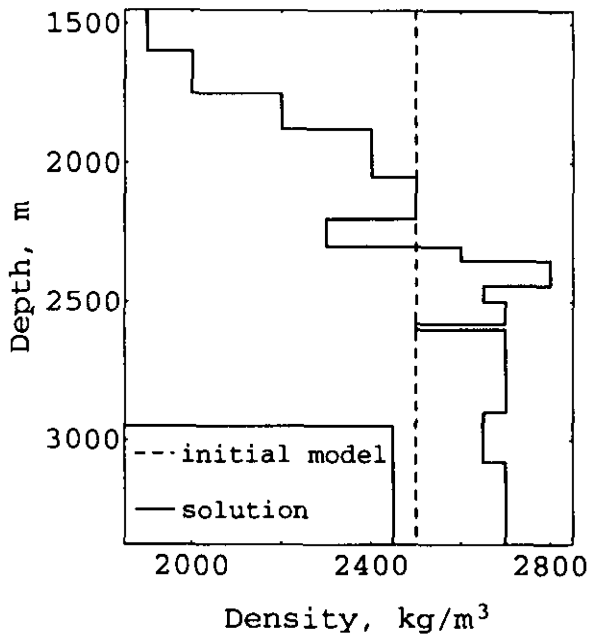

Fig. 4. Amplitude inversion of the synthetic data for density after 5 iterations.

being 2-3 orders of magnitude less than the singular values for density.

A starting model for the density profile was $2.5 \cdot 10^3$ kg/m^3 for all layers. The result of the amplitude inversion for density is shown in Figure 4. The correct values of density for the lower 17 layers (from 1450 to 3375 m) were found in 5 iterations.

Application to Field Data

We used transit times and amplitudes of the vertical components of downgoing P-waves from a two-offset VSP survey to determine the P-wave velocity and density profiles. Two Vibroseis sources were located at 80 m and 1600 m offsets, 6.8 and 8.5 m below the wellhead, respectively. The depth of the vertical well was 3375 m. The seismic signals were recorded using geophones placed in the well at depths of 100 m to 3700 m. We used 75 traces for the near-offset traveltime inversion. The spacing between two adjacent traces was about 150 m in the upper part of the well and about 20 m in the lower part of the well (below 2200 m). For the computations, we considered the medium to be a stack of 25 homogeneous layers with horizontal boundaries. The depths of the boundaries were estimated from the sonic log. The starting model for the P-wave velocity was 3 km/s for all layers.

Figure 5a shows the result of an 80 m offset inversion for P-wave velocity and the corresponding sonic log data. The damping factor was set to 0. The residual was 0.44 ms per trace after 5 iterations. It is known that VSP transit times are greater than sonic log times (De et al., 1994); however, the sonic log times which we used for the comparison were already corrected for dispersion. We applied the chi-square statistic to estimate the fit of the transit times to the model. The standard deviation of the real data was taken as 0.5 ms for all data points. On the final iteration the value of χ^2 was found to be 59. It fell into the range of χ^2 statistic with an average value $\nu = N - K = 50$ and a standard deviation

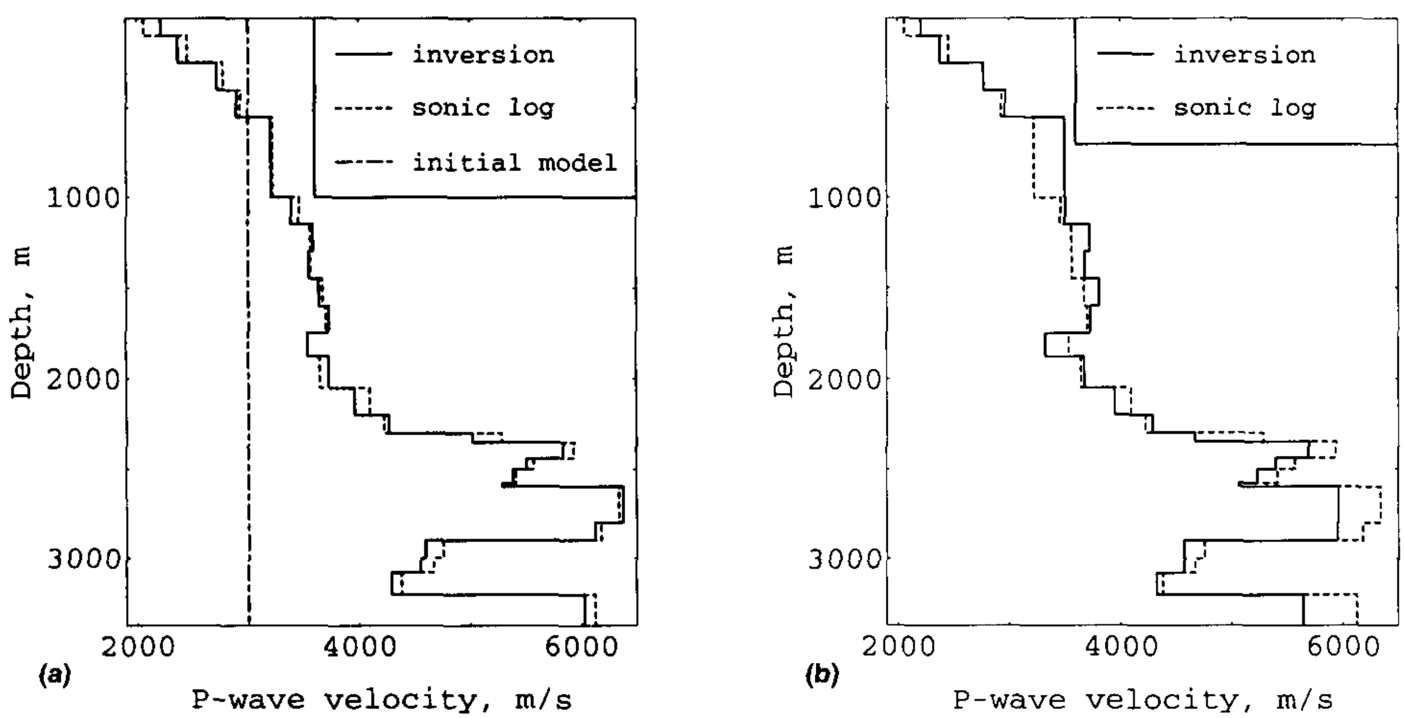

Fig. 5. P-wave velocities from traveltime inversion and average velocities from the sonic log: **(a)** 80 m offset (5 iterations); **(b)** 1600 m offset (3 iterations).

$\sqrt{2\nu} = 10$. The corresponding value of χ^2_ν was 1.18, so the model was considered plausible.

The total number of traces recorded was 90 in the 1600 m offset experiment. The geophones were located from 1500 to 3300 m with the spacing of approximately 20 m. The *P*-wave velocity profile obtained from the near-offset inversion was used as a starting model for the far-offset inversion. The parameter β was chosen to be 10^{-11}. The traveltime inversion of far-offset VSP data is shown in Figure 5b. The final residual was 0.79 ms per trace after 3 iterations. The result was in agreement with both the near-offset inversion and average velocities from the sonic log, though this agreement was worse than in the case of the near-zero offset inversion. On the final iteration $\chi^2_\nu = 225 / 90 = 2.5$. A possible explanation for this fact was that a simple one-dimensional model of a stack of horizontal layers was not completely adequate for the far-offset survey. In particular, the effect of dip may be important. This effect can be taken into account using a two-dimensional generalization of the inversion algorithm.

The amplitude inversion of field data was done for the target zone, i.e., for the lower 12 layers (from 2050 to 3375 m). We used the amplitudes of the vertical components of downgoing *P*-waves from 61 traces for inversion. The top trace was located at 2050 m, the spacing between two adjacent traces was about 20 m, and the bottom trace was at 3370 m. The density log was chosen as a starting model. The parameter β was set to 0. We used the *P*-wave velocity profile obtained from the near-offset traveltime inversion (Figure 5a) to find the transmission coefficients in equation (6) and their partial derivatives. The *S*-wave velocity V_S was assumed to be proportional to the *P*-wave velocity, specifically $V_S = V_P / \sqrt{3}$. The density profile after 3 iterations is

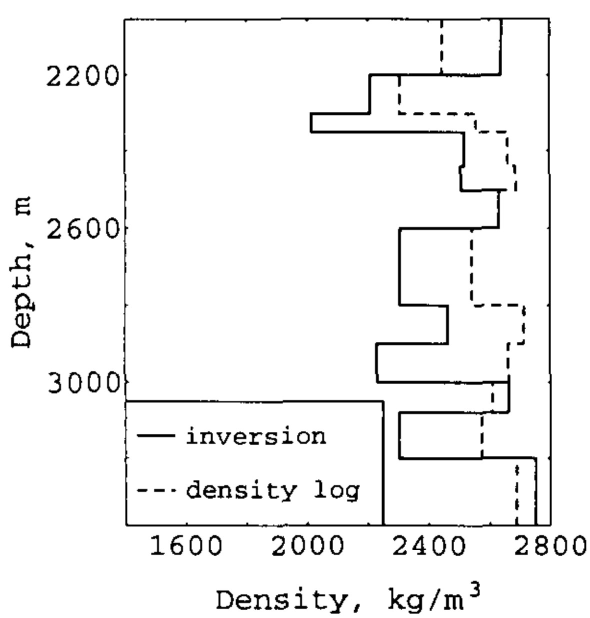

Fig. 6. Densities from inversion of the amplitudes of the vertical components of downgoing *P*-waves and average densities from log data: 80 m offset (3 iterations).

shown in Figure 6. All densities obtained range from 2015 to 2750 kg/m^3. The average log densities vary approximately from 2300 to 2715 kg/m^3.

Conclusions

The linear iterative inversion is a reliable method for finding a *P*-wave velocity profile from the transit time VSP data if a starting model for the velocity is within geological constraints (Stewart, 1984; Pujol et al., 1985). We presented a modification of the method developed earlier (Cheng et al., 1992) to get the density estimates using the vertical amplitudes of the downgoing *P*-waves. This modification was suitable for near-offset VSPs and a stratified, laterally homogeneous model of the earth. Using the results of the traveltime inversion of the near-offset field data, we proved that the region near the well could indeed be described in terms of a one-dimensional model. The same procedure can be applied to the transit times of the *S*-waves to find the *S*-wave velocity variation with depth. A synthetic example was given to show that the density profile could be found from the amplitude information. We also showed that a linear iterative procedure based on the matrix SVD can be used to get estimates of the density from a near-offset VSP experiment.

The results of the amplitude inversion may be improved in several ways. First, a different setup with more traces in the upper part of the well would be beneficial. This should diminish the error in the density determination for the upper layers. In particular, a layer stripping method can be used if the number of observations (transit times or amplitudes) is greater than or equal to the number of unknown model parameters for each layer. Although exact knowledge of the *S*-wave velocity is not necessary for the near-offset density inversion, it is desirable to obtain the *S*-wave velocity estimates and to use them as input for the inversion algorithm to find the density from a far-offset VSP. Finally, one can use the ratio of amplitudes of vertical components of downgoing and upgoing waves rather than the amplitudes of vertical components of downgoing waves in the inversion procedure to diminish the influence of the unknown source function and imperfect coupling of the downhole geophones as it was done by Cheng et al. (1992) for synthetic data.

References

Aki, K. and Richards, P.G., 1980, Quantitative seismology. Theory and methods: W.H. Freeman & Co.

Bevington, P.R., 1969, Data reduction and error analysis for the physical sciences: McGraw-Hill.

Cheng, S., Hron, F. and Daley, P.F., 1992, Determination of shear wave velocities and densities from *P*-wave amplitudes in VSP data: Can. J. Expl. Geophys. **28**, 19-29.

De, G.S., Winterstein, D.F. and Meadows, M.A, 1994, Comparison of *P*- and *S*-wave velocities and *Q*'s from VSP and sonic log data: Geophysics **59**, 1512-1529.

Grivelet, P.A., 1985, Inversion of vertical seismic profiles by iterative modeling: Geophysics **50**, 924-930.

Hron, F. and Kanasewich, E.R., 1971, Synthetic seismograms for deep seismic sounding studies using asymptotic ray theory: Bull. Seis. Soc. Am. **61**, 1169-1200.

Koch, M., 1992, Optimal regularization of the linear seismic inverse problem, *in* Bednar, J.B., Lines, L.R., Stolt, R.H. and Weglein, A.B., Eds., *Geophysical inversion*: Soc. Ind. Appl. Math., 183-244.

Lanczos, C., 1961, Linear differential operators: D. Van Nostrand Co.

Lines, L.R., Bourgeois, A. and Covey, J.D., 1984, Traveltime inversion of offset vertical seismic profiles – a feasibility study: Geophysics **49**, 250-264.

______ and Treitel, S., 1984, Tutorial: a review of least-squares inversion and its application to geophysical problems: Geophys. Prosp. **32**, 159-186.

Macé, D. and Lailly, P., 1986, Solution of the VSP one-dimensional inverse problem: Geophys. Prosp. **34**, 1002-1021.

Marquardt, D.W., 1963, An algorithm for least squares estimation of non-linear parameters: J. Soc. Ind. Appl. Math. **11**, 431-441.

Nolte, B. and Frazer, L.N., 1994, Vertical seismic profile inversion with genetic algorithms: Geophys. J. Internat. **117**, 162-178.

Press, W.H., Teukolsky, S.A., Wetterling, W.T. and Flannery, B.P., 1992, Numerical recipes in FORTRAN: the art of scientific computing: Cambridge Univ. Press.

Pujol, J., Burridge, R. and Smithson, S., 1985, Velocity determination from offset vertical seismic profiling data: J. Geophys. Res. **90**, 1871-1880.

Shahriar, M., 1986, Application of iterative inversion techniques in crustal seismology: Ph.D. thesis, Univ. of Alberta.

Stewart, R.R., 1984, VSP interval velocities from traveltime inversion: Geophys. Prosp. **32**, 608-628.

Zelt, C.A. and Smith, R.B., 1992, Seismic traveltime inversion for 2-D crustal velocity structure: Geophys. J. Internat. **108**, 16-34.